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Filter bank property of variational mode decomposition and its applications

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ABSTRACT

The variational mode decomposition (VMD) was proposed recently as an alternative to the empirical mode decomposition (EMD). To shed further light on its performance, we analyze the behavior of VMD in the presence of irregular samples, impulsive response, fractional Gaussian noise as well as tones separation. Extensive numerical simulations are conducted to investigate the parameters mentioned in VMD on these filter bank properties. It is found that, unlike EMD, the statistical characterization of the obtained modes reveals a different equivalent filter bank structure, robustness with respect to the non-uniformly sampling and good resolution in spectrum analysis. Moreover, we illustrate the influences of the main parameters on these properties, which provides a guidance on tuning them. Based on these findings, three potential applications in extracting time-varying oscillations, detrending as well as detecting impacts using VMD are presented.

1. Introduction

Signal and data analysis is an important and necessary part in both research and practical applications. The essential information in the data is often mingled together with other irrelevant information. The primary goal of signal processing is to capture underlying information and structures, e.g. impacts and trends, which are often challenges due to the nonlinear and nonstationary nature of the data. Fourier and wavelet transforms correspond to the use of some basis (or frame), predefined independently of the processed signal, while adaptive methods can construct such a basis directly based on the information contained in the signal. Empirical mode decomposition (EMD), pioneered by Huang et al. [2], is a data-driven algorithm to represent nonstationary signals as sums of zero-mean intrinsic mode functions (IMFs). EMD essentially represents the signal as an expansion of basis functions that are signal-dependent via the recursive sifting iterative procedure, unlike the Fourier and wavelet transform. The EMD method has gained a lot of interest in signal analysis in this last decade, because of its usefulness in separating stationary and non-stationary components from a signal. However, EMD remains an exclusively empirical algorithm, without a solid mathematical foundation, despite numerous attempts so far made to improve the understanding of the way it operates or to enhance its performance. For instance, it has been demonstrated that IMFs obtained by EMD provide frequency responses similar to that of a dyadic filter bank [3]. On the other hand, some novel adaptive methods have been developed with a more firm theoretical foundation, e.g. the synchrosqueezing transform formulated by Daubechies et al. in [4]. The stability and robustness properties of the synchrosqueezing transform to bounded perturbations of the

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signal and to Gaussian white noise have been also investigated in [5]. Hou et al., however, proposed a partially variational approach to the traditional EMD, in which the signal is recursively decomposed into an IMF with TV3-smooth envelope and a TV3-smooth residual [6]. Moreover, the empirical wavelet transform (EWT) is a relatively recently proposed algorithm, which explicitly builds an adaptive wavelet basis to expand a given signal into adaptive subbands [7].

VMD is a very newly developed methodology as an alternative to the EMD method, which can adaptively decompose a multi-component signal into a number of quasi-orthogonal intrinsic mode functions [1]. It has been verified VMD outperforms EMD with regards to tone detection and separation as well as noise robustness [1]. However, little is known, indeed, on the decomposition achieved by VMD when analyzed signals are only the realization of some stochastic process, along with the effects of parameters mentioned in VMD on these equivalent filters. Noisy or nonuniformly sampled data are ubiquitous in engineering and natural science, hence the stability and robustness of VMD should be also investigated in such cases. In addition, it would be rather interesting to study the behavior of VMD for the problem of tones separation and to see whether it exhibits a similar "beating" phenomenon. The feature of tones separation for the VMD have been partly investigated in [1], nevertheless the influence of the main parameters on this performance is still an open question.

The aim of this paper is to deal with the above delineated issues of the VMD with respect to nonuniform samples, equivalent impulse response and filter bank structure, as well as tones-separation. Considering key parameters mentioned in VMD are expected to affect the resulting behaviors and the decomposed components to some extent, hence this paper attempts to address in a well-controlled (hopefully) use of the VMD technique for practical applications. More precisely, an overview of VMD is first given in Section 2. In Section 3, we investigate four inherent features of VMD through extensive numerical experiments. Subsequently, three potential applications of VMD are illustrated in Section 4. Discussion and conclusions are presented in Sections 5 and 6, respectively.

2. Variational mode decomposition

VMD can non-recursively decompose a real-valued multi-component signal f into a discrete number of quasi-orthogonal band-limited sub-signals u_k with specific sparsity properties in the spectral domain [1]. Each mode is compact around a center pulsation ω_k and its bandwidth is estimated using $\mathcal{H}^1$ Gaussian smoothness of the shifted signal. For the convenience of following discussions, let us commence by calling these modes obtained by VMD as band-limited IMFs (BLIMFs) in this work. BLIMFs are different from IMFs defined in EMD technique according to the numbers of extrema and zerocrossings. The VMD technique is essentially written as a constrained variational problem in [1]:

$$\min_{\{u_k\},\{\omega_k\}} \left\{ \sum_{k=1}^{K} \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\omega_k t} \right\|_2^2 \right\}$$

$$\text{subject to} \quad \sum_{k=1}^{K} u_k = f \tag{1}$$

where u_k is the decomposed BLIMF and K is known a priori. The constraint in (1) can be addressed by introducing a quadratic penalty and Lagrangian multipliers. The augmented Lagrangian is thus given as follows:

$$\mathcal{L}(\{u_k\},\{\omega_k\},\lambda) = \alpha \sum_{k=1}^{K} \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\omega_k t} \right\|_2^2 + \left\| f(t) - \sum_{k=1}^{K} u_k(t) \right\|_2^2 + \left\langle \lambda(t), f(t) - \sum_{k=1}^{K} u_k(t) \right\rangle \tag{2}$$

The above (2) defines an augmented Lagrangian, and the saddle point of (2) corresponding to the solution (1) is found using Alternate Direction Method of Multipliers (ADMM) [8]. All the modes and center frequencies, gained from the solution in Fourier domain, are updated in two directions. The estimate of the k-th mode is updated using

$$\hat{u}_k^{n+1}(\omega) = \frac{\hat{f}(\omega) - \sum_{i<k} \hat{u}_i^{n+1}(\omega) - \sum_{i>k} \hat{u}_i^{n}(\omega) + \frac{\hat{\lambda}^n(\omega)}{2}}{1 + 2\alpha(\omega - \omega_k^n)^2} \tag{3}$$

where α is known as the balancing parameter of the data-fidelity constraint. Wiener filtering, hence, is potentially embedded in the VMD algorithm, which makes it much more robust to sampling and noise. The center frequency ω_k is updated as the center of gravity of the corresponding positive part of mode's power spectrum, which can be represented by

$$\omega_k^{n+1} = \frac{\int_0^\infty \omega \left| \hat{u}_k^{n+1}(\omega) \right|^2 d\omega}{\int_0^\infty \left| \hat{u}_k^{n+1}(\omega) \right|^2 d\omega} \tag{4}$$

As far as initializations are concerned, two simple and fundamental methods for setting $\omega_k^0, k = 1, \dots, K$, are considered in this work, namely, uniformly spaced distribution and zero initial. Equivalently, the initialization of center frequencies with uniformly spaced distribution is defined as

$$\mathcal{P}_{\mathcal{U}}: \omega_k^0 = \frac{k-1}{2K}, \quad k = 1, \dots, K \tag{5}$$

while the zero initial for the center frequencies is denoted as

$$\mathcal{P}_{\mathcal{Z}}: \omega_k^0 = 0, \quad k = 1, \dots, K. \tag{6}$$

It is not the purpose of this paper to reintroduce VMD algorithm in detail. The interested readers are referred to [1] for complete algorithm and the implementation of the VMD. Nevertheless, the reliance of the initialization of center frequencies of the modes on the decomposition, and the particular choice of α for a specific application, will be investigated at length in the following sections.

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