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Fast and accurate estimator for two-dimensional cisoids based on QR decomposition

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ABSTRACT

In this paper, we apply the QR decomposition to parameter estimation for K two-dimensional (2-D) complex-valued sinusoids embedded in additive white Gaussian noise. By exploiting the rank-K and linear prediction (LP) properties of the 2-D noise-free data matrix, we show that the frequencies and damping factors of one dimension can be extracted from the first K rows of $\mathbf{R}$, that is, the upper triangular matrix in the factorization. An iteratively weighted least squares (IWLS) algorithm is then proposed to estimate the LP coefficients from which the sinusoidal parameters in this dimension are computed. The frequencies and damping factors of the remaining dimension are estimated using a similar IWLS procedure such that the 2-D parameters are automatically paired. We thus refer our estimator to as the QR-IWLS algorithm. Moreover, we have analyzed its bias and mean square error performance. In particular, closed-form expressions are derived for the special case of $K=1$. The performance of the QR-IWLS method is also evaluated by comparing with several state-of-the-art 2-D harmonic retrieval algorithms as well as Cramér–Rao lower bound via computer simulations.

1. Introduction

Many important problems in signal processing can be formulated as sinusoidal parameter estimation where the frequencies, damping factors and/or amplitudes are required to determine from noisy observations. For the one-dimensional (1-D) case, application areas include communications, radar, sonar, speech analysis, astronomy, control theory, instrumentation and measurement [1–8], and more recently, smart grid [9]. Apart from the commonly considered 1-D parameter estimation, multi-dimensional sinusoids are also useful in modeling many real world signals. For example, the sinusoidal parameters of 2-D observations can correspond to the position information of multiple targets in wireless communication channel [10] and multiple-input multiple-output radar [11,12]. Moreover, the measurements in nuclear magnetic resonance spectroscopy [13], which is a powerful technique for protein research in food and nutritional industries, can be modeled as a sum of multi-dimensional damped sinusoids where the frequencies and damping factors are crucial to determine the protein structures.

Given the 2-D observations of K cisoids embedded in Gaussian distributed disturbances, the maximum likelihood (ML) method [14] can provide the optimum parameter estimation performance in the sense that its mean square error (MSE) attains the Cramér–Rao lower bound (CRLB). However, its computational complexity is quite demanding because we need to find the global maximum in the ML cost function, which is nonlinear with $2K$ variables. To reduce the computation, relaxation can be made in the ML formulation to produce a near-optimum solution

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[14,15]. Nevertheless, the subspace approach is more popular than the ML based methods because it is more computationally attractive and is able to achieve a very high estimation accuracy. The core idea of the subspace methodology is to separate the observations into the so-called signal and noise subspaces via eigenvalue decomposition of the sample covariance matrix or singular value decomposition (SVD) of the received data matrix from which the sinusoidal parameters are determined using the resultant eigenvalues, eigenvectors, singular values and/or singular vectors. Here, the K largest eigenvalues and singular values and their associated eigenvectors and singular vectors, respectively, correspond to the signal subspace while the remaining components belong to the noise subspace. Multiple signal classification (MUSIC) [11,13], and estimation of signal parameters via rotational invariance techniques (ESPRIT) [16–18], are two well-known and conventional subspace algorithms, but they generally provide suboptimal estimation performance. Recently, we have proposed a subspace method called principal-singular-vector utilization for modal analysis (PUMA) [19,20], whose derivation is founded by the knowledge that the K left and right principal singular vectors contain the frequencies and damping factors in the first and second dimensions, respectively. By making the most of these $2K$ vectors, the MSE of the PUMA algorithm can meet the CRLB at sufficiently small noise and/or large data size conditions. To avoid performing SVD, an even faster 2-D frequency estimator [21] has been devised by exploiting the correlation of the data samples, whose MSE approaches the CRLB when the noise power tends to zero and/or the data size tends to infinity. However, this technique is designed only for a single undamped multi-dimensional cisoid. As SVD is required in [19,20], we are motivated to explore other matrix factorizations which are more computationally attractive to perform the 2-D sinusoidal parameter estimation. In this work, replacing the SVD by QR decomposition is investigated. Unlike the PUMA method where the frequency and damping factor information is contained in the left and right dominant principal singular vectors, sinusoidal parameters of only one dimension are embedded in the QR decomposition. Although both PUMA and the proposed schemes are based on an iteratively weighted least squares (IWLS) procedure, our weighting matrix is simpler, and only one iteration is required for parameter convergence. To the best of our knowledge, application of QR factorization to 2-D sinusoidal parameter estimation has not been studied in the literature, although it has been addressed for the 1-D case [22,23].

The rest of the paper is organized as follows. In Section 2, the 2-D sinusoidal parameter estimation problem is first formulated. Then we show that the frequency and damping factor information of one dimension is contained in the first K rows of $\mathbf{R}$, namely, the upper triangular matrix in the QR factorization of the observed data matrix. By utilizing the linear prediction (LP) property and weighted least squares (WLS) technique, an iterative procedure that operates on these K rows is devised for estimating the frequencies and damping factors in this dimension. By exploiting an alternative decomposition of the received data matrix, the frequency and damping factor parameters of the remaining dimension are also estimated using a similar IWLS procedure such that 2-D frequency pairing is automatically achieved. The proposed estimator is referred to as the QR-IWLS algorithm. In Section 3, the mean and MSE expressions of the proposed estimator are produced. Numerical examples are included in Section 4 to corroborate the theoretical calculations and to evaluate the estimation performance of the QR-IWLS algorithm by comparing with the ML [14], ESPRIT [17], and PUMA [20] schemes as well as the CRLB. Finally, conclusions are drawn in Section 5.

Notation: Scalars, vectors and matrices are denoted by italic, bold lower-case and bold upper-case symbols, respectively. The angle and magnitude of a are represented as $\angle(a)$ and $|a|$. The derivative of a function $f(a)$ with respect to a is $f'(a)$. The noise-free $\mathbf{A}$ is denoted by $\tilde{\mathbf{A}}$, its estimate is $\hat{\mathbf{A}}$ and its rank is $\mathrm{rank}(\mathbf{A})$, while ^{T}, ^{H}, $*$, $^{-1}$ and † are the transpose, conjugate transpose, complex conjugate, inverse, and pseudoinverse operators, respectively. The mth element of $\mathbf{a}$ is $[\mathbf{a}]_m$ and the (m,n) entry of $\mathbf{A}$ is either denoted by $[\mathbf{A}]_{m,n}$ or $a_{m,n}$. The $\mathbf{I}_M$ symbolizes the $M \times M$ identity matrix and $\mathbf{0}_{M\times N}$ represents the $M \times N$ zero matrix. The $\mathrm{diag}(\mathbf{a})$ denotes a diagonal matrix with vector $\mathbf{a}$ as the main diagonal and $\mathrm{Toeplitz}(\mathbf{a}, \mathbf{b}^T)$ is a Toeplitz matrix with first column $\mathbf{a}$ and first row $\mathbf{b}^T$. The vec is the vectorization operator, $\mathbb{E}$ is the expectation operator, and $\otimes$ represents the Kronecker product. Finally, $\mathbb{R}$ and $\mathbb{C}$ are used to represent the sets of real and complex numbers, respectively.

2. Algorithm development

We first state the 2-D sinusoidal parameter estimation problem as follows. The observed data are

$$x_{m,n} = s_{m,n} + w_{m,n}, \quad m = 1,2,\ldots,M, \quad n = 1,2,\ldots,N, \tag{1}$$

where

$$s_{m,n} = \sum_{k=1}^{K} \gamma_k \alpha_k^m \beta_k^n \exp\{j(\mu_k m + \nu_k n)\}, \tag{2}$$

is the noise-free signal which contains $K \geq 2$ damped cisoids. Note that the simplest case of $K=1$ will also be discussed at the end of this section. The $\gamma_k \in \mathbb{C}$ is the complex-valued amplitude, $\mu_k \in (-\pi, \pi)$ is the frequency and $\alpha_k \in \mathbb{R}$ is the damping factor, in the first dimension, while $\nu_k \in (-\pi, \pi)$ and $\beta_k \in \mathbb{R}$ are the corresponding parameters in the second dimension, of the kth tone. All $\{\gamma_k\}$, $\{\mu_k\}$, $\{\alpha_k\}$, $\{\nu_k\}$ and $\{\beta_k\}$ are unknown deterministic constants. On the other hand, $w_{m,n}$ represents the additive zero-mean noise which is modeled as a complex white Gaussian process with unknown variance σ^2. The dimensions of the 2-D observations are $M \times N$. It is assumed that K is known *a priori* and $\min(M,N) > K$. We further assume that the frequencies in at least one of the dimensions are distinct. Our objective is to find the frequencies and damping factors from the MN measurements of $\{x_{m,n}\}$. Note that once these parameters have been estimated, the amplitudes can be easily determined as they are linear in (2).