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Estimating the parameters of ultrasonic echo signal in the Gabor transform domain and its resolution analysis

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ABSTRACT

The accurate estimation of parameters of an ultrasonic signal is essential in ultrasonic non-destructive testing (NDT). In this paper, a novel method for parameter estimation based on the Gabor transform (GT) is proposed. Firstly, the GT is used in ultrasonic signal processing in order to estimate time of flight (TOF) and center frequency (CF) by calculating where maximum modulus of Gabor transform function of the signal is attained. The quasi-Newton method is employed to improve the estimation accuracy and reduce computational complexity. The Gabor transform function of the signal has an analytic expression which contributes to derive other estimators relating to TOF and CF such as bandwidth factor, phase and amplitude. The parameters of multi-component echo signal are estimated by estimating the parameters of each component successively. Numerical simulation has been carried out to show the performances of the proposed method in estimating parameters of ultrasonic signal.

1. Introduction

Most materials can be penetrated by ultrasound, which is usually used to detect the internal and surface defects of these materials, and evaluate their physical and mechanical properties. The ultrasonic detection technique uses relatively simple equipment, without injury to persons, low-cost and easy to implement modern information processing and computer automatic control, etc. Therefore, it is widely applied in NDT technology [1–4].

The ultrasonic signal processing method is based on estimating the parameters of ultrasonic echo signal such as TOF, CF, phase and amplitude. Methods for estimating TOF or CF are usually the cross-correlation method and its improved versions [5–7], MUSIC [8] and CAPON [9]. Because the ultrasonic echoes usually contain noise, the spectral analysis methods using discrete Fourier transform, tend to have high uncertainty, and result in large errors [10,11]. The cross-correlation method is optimal when the following conditions are met [12]: (1) the received signal is embedded in White Gaussian Noise (WGN), and (2) the echo signal is the time-shifted, amplitude-scaled replica of a reference signal. If any of the above conditions is not satisfied, it will affect the estimation accuracy. Demirli and Saniie [12] have introduced the modeling and parameter estimation of the ultrasonic echo using the maximum likelihood estimation (MLE) and the expectation maximization (EM) algorithm. However, the implementations are potentially complicated. Number of iterations for estimating two closely spaced overlapping echoes with a SNR of 11.7 dB is 889. The approaches based on the short-time Fourier transform (STFT) are presented for estimating parameters of ultrasound [15,16], but the choice of window function affects estimation accuracy.

In this study, an estimation algorithm for ultrasonic echo signal based on GT is presented according to the characteristics of time-frequency analysis in Gabor transform domain. GT is a classical time-frequency representation tool which allows one

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to obtain localized information of time and frequency of a signal, and it has been used for parameter estimation [17], denoising of signal [18,19] and signal decomposition [20,21]. Although in many aspects the Wigner–Ville distribution (WVD) is superior to the GT spectrogram, its applications are limited by the existence of cross-term interference. Compared with WVD, the GT has the advantages of (1) avoiding the cross-term problem and (2) consuming less computation time [19]. Furthermore, the Hough transform also provides significant suppression of cross-terms in the multicomponent case [22]. It is only suitable for linear or locally linear FM signals. Furthermore, there is also a disadvantage in the windowing used in the pseudo-Wigner–Hough transform (PWHT). The noise floor has increased due to the shorter summation interval used, which affects estimation accuracy [22]. Wavelet transform is also a useful tool of linear time-frequency representation, which is applied in ultrasonic flaw detection [23]. However, it is difficult to select wavelet kernels and the number of scales, which are related to accuracy. In [24], Newton's method is applied to improve the accuracy and reduce the computational burden for estimating TOF and CF. However, depending on the initial value, Newton's method is not guaranteed to converge to the global maximum point. The quasi-Newton methods that build up an approximation of the inverse Hessian are often regarded as the most sophisticated for solving unconstrained problems. Therefore, the quasi-Newton method is employed in our study.

This paper is organized as follows. The GT is described in Section 2. Section 3 presents resolution analysis of the time domain and the frequency domain. Section 4 describes the algorithm of parameter estimation for ultrasonic echo signal. The performance of the estimation through CRLB is analyzed in Section 5. In Section 6, numerical simulation results are obtained in order to evaluate the performance of the proposed algorithms.

2. Gabor transform of ultrasonic echo signal

For a continuous time signal $x(t)$, the Gabor transform or analysis transform, $CGT: L^2(R) \rightarrow L^2\left(R^2\right)$ is defined by [25]

$$G_x(\omega,\tau) = \int_{-\infty}^{\infty} x(t)\gamma^*(t-\tau)\exp(-j2\pi\omega t)dt = \int_{-\infty}^{\infty} x(t)\gamma^*_{\omega,\tau}(t)\,dt \tag{1}$$

where, $\gamma_{\omega,\tau}(t) = \gamma(t-\tau)\exp(j2\pi\omega t)$, $\gamma(t)$ is an analysis window function and $\gamma^*(t)$ denotes the complex conjugate of $\gamma(t)$. According to Heisenberg's uncertainty principle, the Gaussian window of GT has the property of optimal concentration and, therefore, is useful for time-frequency analysis. Both the time and frequency resolution simultaneously cannot be made infinitely small according to Heisenberg's uncertainty principle, because of a lower bound on the time-bandwidth product,

$$\Delta t\ \Delta f \geq \frac{1}{4\pi} \tag{2}$$

where, Δt and Δf are defined as time centroid and frequency centroid, respectively.

The equality sign of inequality (2) is obtained by using Gaussian modulated pulse [26]. In this study, Gaussian function with unit energy $\gamma(t) = 2^{1/4}e^{-\pi t^2}$ is used as an analysis window in the Gabor transform and, therefore, Gabor basis function $\gamma_{\omega,\tau}(t) = \gamma(t-\tau)\exp(j2\pi\omega t)$ has the property of optimal concentration in the joint time-frequency plane.

GT is a linear time-frequency operator. For $x(t) = \sum_{k=1}^{M} x_k(t)$, the GT of $x(t)$ is expressed as

$$G_x(\omega,\tau) = \sum_{k=1}^{M} G_{x_k}(\omega,\tau) \tag{3}$$

where $G_{x_k}(\omega,\tau)$ is the GT of $x_k(t)$, $k=1\ldots M$.

Let $x(k)$ be a discrete time series with a length L , then its GT can be defined as [27]

$$C_{m,n} = \sum_{k=0}^{L-1} x(k)\gamma^*(k-m\ \Delta M)\ \exp\left(\frac{-j2\pi n\ \Delta Nk}{L}\right) \tag{4}$$

where, ΔM and ΔN are time and frequency sampling intervals, respectively. $L = M(\Delta M) = N(\Delta N)$, M and N are the numbers of sampling points in time and frequency domains, respectively. An oversampling rate is defined as $\eta = MN/L \geq 1$.

In pulse-echo ultrasonic testing, the backscattered echo from a flat surface reflector can be modeled as [12]:

$$s(t) = \beta\ \exp\left(-\alpha(t-\tau)^2\right)\ \cos\left(2\pi f_c(t-\tau)+\phi\right) \tag{5}$$

The parameters of this model are closely related to the physical properties of the material through which the ultrasonic signal propagates. The TOF τ is related to the distance between the transducer and the reflector. β is the amplitude parameter of the signal accounting for the attenuation and the size of the reflector. f_0 and α are the center frequency and bandwidth factor respectively. The phase of the signal ϕ accounts for the orientation of the reflector.

The complex model is written as

$$z(t) = \beta\ \exp\left(-\alpha(t-\tau)^2 + j\left(2\pi f_c(t-\tau)+\phi\right)\right) \tag{6}$$

The Gabor transform of the complex model (6) can be derived as

$$\begin{aligned} G_z(\omega,t) = 2^{1/4}\beta \int_{-\infty}^{\infty} &\exp\big(-(\alpha+\pi)u^2 + 2(\alpha\tau+\pi t+j\pi f_c - j\pi\omega)u \\ &-\left(\alpha\tau^2+\pi t^2+j2\pi f_c\tau - j\phi\right)\big)\ du, \end{aligned} \tag{7}$$

and further simplified to be

$$\begin{aligned} G_z(\omega,t) = 2^{1/4}\beta\sqrt{\frac{\pi}{\alpha+\pi}}\exp\Bigg[&-\frac{1}{(\alpha+\pi)}\left(\alpha\pi(\tau-t)^2+\pi^2(f_c-\omega)^2\right) \\ &+j\left(\frac{2\pi(\pi t+\alpha\tau)(f_c-\omega)}{(\alpha+\pi)} - (2\pi f_c\tau-\phi)\right)\Bigg] \end{aligned} \tag{8}$$

Thus the Gabor spectrum or the magnitude of the $G_z(\omega,t)$ which is used for estimation of the signal parameters can be derived as

$$\left|G_z(\omega,t)\right| = 2^{1/4}\beta\sqrt{\frac{\pi}{\alpha+\pi}}\exp\left[-\frac{1}{(\alpha+\pi)}\left(\alpha\pi(\tau-t)^2+\pi^2(f_c-\omega)^2\right)\right]. \tag{9}$$

The maximum of (9) can be obtained by taking partial derivatives of $\left|G_z(\omega,t)\right|$ with respect to variables ω and t: $\partial\left|G_z(\omega,t)\right|/\partial\omega = 0$, $\partial\left|G_z(\omega,t)\right|/\partial t = 0$, and leads to an estimation of CF and TOF:

$$\omega = f_c, \quad t = \tau. \tag{10}$$