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Brief paper

Model predictive control of linear systems over networks with data quantizations and packet losses☆

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ABSTRACT

This paper addresses the synthesis approach of model predictive control (MPC) for the constrained linear systems under networked environment where both data quantization and packet loss may occur. Based on a previous approach which considers packet loss only, this paper proposes an extended approach which incorporates data quantization effect. Technically, the infinite horizon control moves are parameterized into a single state feedback law, at each sampling instant, and the robust stability is specified for all possible packet losses and quantization errors. Both the closed-loop model and the resultant optimization problem include the previous packet-loss-only results as special cases. A numerical example is given to illustrate the effectiveness of the developed MPC.

1. Introduction

Many control systems are remotely implemented via communication networks with limited bandwidth. Such control systems are commonly referred to as networked control systems (NCSs) (Baillieul & Antsaklis, 2007; Yue, Peng, & Tang, 2006; Zhang, Branicky, & Phillips, 2001). The quantization of state and/or input signals is a peculiar characteristic in NCSs. It is well known that data quantization has an undesirable effect on system performance or even stability, and therefore a lot of works have been carried out to analyze and mitigate the effect. Under the assumption that the quantizer is static and time-invariant, Elia and Mitter (2001) addresses that the quantizer needs to be logarithmic in quadratic stabilization of single-input–single-output (SISO) discrete linear time-invariant (LTI) system. In Elia and Mitter (2001), the optimal logarithmic quantizer is given explicitly in terms of the unstable eigenvalues of the system. Fu and Xie (2005) generalizes the results of Elia and Mitter (2001) to the multi-input–multi-output (MIMO) systems and the output feedback control, and converts the quantized quadratic stabilization problem into the robust control problem by using the sector bound method. Based on the results in Fu and Xie (2005), a simple dynamic scaling method for a logarithmic quantizer based output feedback controller is given in Fu and Xie (2009). These papers consider only one quantizer existing in one communication link, either in the sensor to the controller (S–C) link or in the controller to the actuator (C–A) link. For the NCSs with quantizers both in S–C and C–A links, the stabilization problem has been discussed in Coutinho, Fu, and de Souza (2010) and Picasso and Bicchi (2007) recently.

The packet loss behavior is another important issue and one of the potential sources of instability and poor performance in NCSs. There have been many interesting studies on packet loss issue. In Xiong and Lam (2007), the stabilization problem of NCS with packet losses in both links is addressed, where bounded packet loss process and Markovian packet loss process are considered. Zhang and Yu (2007) investigates the design of the observed-based output feedback control for NCS with packet loss, where the NCS model is described as a discrete-time switched system with four subsystems. Ding (2011) and Ding and Tang (2010) address the synthesis approaches of MPC for NCS with packet losses in both links.

It is common and true of some real NCSs that the effects of the data quantization and packet loss are coexisting. Most recently, the stabilization problem for NCSs when considering both the data quantization and packet loss has been provided in Xiao, Xie, and Fu (2010) and You and Xie (2011). By applying mode-dependent quantized state feedback, Xiao et al. (2010) investigates the stabilization problem for single-input Markov jump linear systems.

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You and Xie (2011) addresses the minimum data rate for mean square stabilizability of linear systems over an undesirable communication link. However, both works consider that the quantizer and packet loss only exist in one of the links.

MPC is a popular technique for the control of industrial process. A synthesis approach of MPC is the one which guarantees the closed-loop stability, i.e., the closed-loop system is stable whenever the optimization problem is feasible. The unique feature of MPC is the ability to handle the physical constraints in a systematic manner. Hence, it is necessary and interesting to generalize the synthesis approach of MPC to the network environment. Although there are many nice papers considering the design of MPC for NCSs (see e.g. Quevedo & Neŝić, 2012, Quevedo, Ostergaard, & Neŝić, 2011, Xue, Li, Li, & Zhu, 2010, Zhao, Liu, & Rees, 2008), only limited works have been found on the synthesis approach of MPC. There are mainly two difficulties in extending the synthesis approach of MPC to the networked environment. Firstly, although many NCS models have been addressed, at present there is no effective NCS model for the synthesis approach of MPC. Secondly, for non-networked MPC, the closed-loop stability can be guaranteed by imposing appropriate constraints in the optimization problem (see e.g. Kothare, Balakrishnan, & Morari, 1996). However, when communication networks are taken into consideration, these constraints do not maintain the desired closed-loop stability.

Unlike the existing works for NCS with logarithmic quantizers and/or packet losses, this paper targets to give a synthesis approach of MPC for constrained discrete LTI MIMO system over networks where both data quantization and packet loss in both links may occur. In this paper, we generalize the results in Ding (2011) and Ding and Tang (2010), where the synthesis approaches of MPC for NCS with packet losses are investigated, to a more complex networked environment. By combining the sector bound approach in Fu and Xie (2005) and the model of NCS with packet loss in Ding (2011) and Ding and Tang (2010), the NCS model is constructed for the synthesis approach of MPC that describes both the data quantization and the packet loss in a unified framework. Furthermore, based on the developed new model, a robust MPC is provided, which guarantees the exponential stability of the closed-loop system. There are mainly two differences between this paper and (Ding, 2011; Ding & Tang, 2010). Firstly, the model in this paper accommodates the data quantization and the packet loss simultaneously, while the model in Ding (2011) and Ding and Tang (2010) describes the packet loss only. Secondly, due to the effect of data quantization, all the details of the synthesis approach of MPC are changed, as compared with (Ding, 2011; Ding & Tang, 2010), especially for the treatment of the initial condition on the augmented state. In conclusion, the results in this paper incorporates the results in Ding (2011) and Ding and Tang (2010) as special cases.

Notation: I is the identity matrix with appropriate dimension. For any vector x and matrix W, $\|x\|_W^2 := x^{\mathrm{T}}Wx$. The superscript T denotes the transpose for vectors or matrices. $x(k+i|k)$ is the value of vector x at a future time $k+i$ predicted at time k. The symbol $(*)$ is used to induce symmetric matrices. A value with superscript $*$ corresponds to the optimal solution of the optimization problem.

2. Modeling of NCS

The framework of NCS considered in this paper is illustrated in Fig. 1, where $u \in \Re^m$ and $x \in \Re^n$ are input and measurable state, respectively. S_{act} is the historical transmission status in C–A link. $S_{\mathrm{act}}(k) = [s_{\mathrm{act}}(k-1), s_{\mathrm{act}}(k-2), \ldots, s_{\mathrm{act}}(k-d_1)]$, where $k \geq 0$ is the sampling instant and d_1 is the upper bound of packet loss in S–C link which will be discussed later. If the actuator receives new data at $k-i$, then $s_{\mathrm{act}}(k-i) = 1$; otherwise, $s_{\mathrm{act}}(k-i) = 0$. $w \in \Re^n$ and S_{act} are the inputs of the controller, while $v \in \Re^m$ is the output of the controller. The "Plant" model in Fig. 1 is

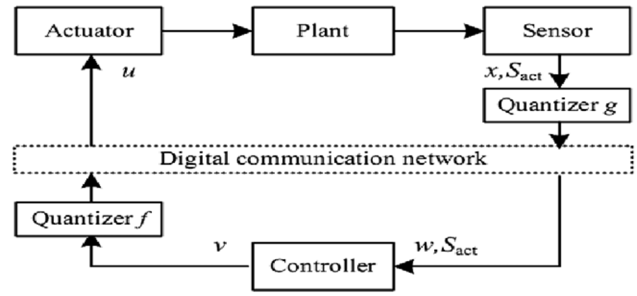

Fig. 1. The structure of the NCS.

$$x(k+1) = Ax(k) + Bu(k), \quad k \geq 0, \tag{1}$$

where A and B are constant matrices of appropriate dimensions. The "Sensor" and the "Actuator" in Fig. 1 are modeled by the identity.

The following assumptions for the NCS are made throughout the paper:

- A1 The sensor, the controller and the actuator are time-synchronized.
- A2 The sensor is clock driven, i.e., it sends $\{x, S_{\mathrm{act}}\}(k)$ at each k. The controller is event-driven, i.e., it calculates v if and only if it receives a new w. The actuator is event-driven, i.e., it updates if and only if it receives a new u.
- A3 In the S–C link, a single packet of the data set $\{x, S_{\mathrm{act}}\}(k)$ is sent at each k, which is subject to quantization and possibility of packet loss. However, since each element of S_{act} is either 0 or 1, the quantizer will not change the value of S_{act}.
- A4 In the C–A link, a single packet of the data $v(k)$ is sent at each k, which is subject to quantization and possibility of packet loss.
- A5 If the actuator does not receive data at k, then the control input at time $k-1$ is applied (zero-order hold effect).

The information S_{act} is essential for the proof of recursive feasibility of the resultant MPC, which will be clear in the sequel.

2.1. Quantizers description

According to assumptions A3 and A4, both x and v are quantized before sent into the network. The two quantizers are modeled by

$$u(k) = f(v(k)), \tag{2}$$
$$w(k) = g(x(k)), \tag{3}$$

where $f(v(k)) = [f(v_1(k))\, f(v_2(k)) \cdots f(v_m(k))]^{\mathrm{T}}$, $g(x(k)) = [g(x_1(k))\, g(x_2(k)) \cdots g(x_n(k))]^{\mathrm{T}}$, and $f(\cdot)$ and $g(\cdot)$ are logarithmic quantizers. In terms of Fu and Xie (2005), $f(\cdot)$ and $g(\cdot)$ satisfy

$$\zeta(\theta) = \begin{cases} \zeta_s, & \text{if } \dfrac{1}{1+\delta_\zeta}\zeta_s < \theta \leq \dfrac{1}{1-\delta_\zeta}\zeta_s,\ \theta > 0, \\ 0, & \text{if } \theta = 0, \\ -\zeta(-\theta), & \text{if } \theta < 0, \end{cases}$$

where ζ is f or g, and δ_ζ is a known number satisfying $0 < \delta_\zeta < 1$. More details for the quantizer can be found in Elia and Mitter (2001), Fu and Xie (2005) and are skipped here for brevity.

By using the sector bound approach in Fu and Xie (2005), $f(v(k))$ and $g(x(k))$ can be written as

$$f(v(k)) = (I + \Delta_f(k))v(k), \tag{4}$$
$$g(x(k)) = (I + \Delta_g(k))x(k), \tag{5}$$