Automatica 49 (2013) 1392–1396

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Identification of systems with localised nonlinearity: From state-space to block-structured models ☆

Anne Van Mulders [1], Johan Schoukens, Laurent Vanbeylen

Vrije Universiteit Brussel, Department ELEC, Pleinlaan 2, B1050 Brussels, Belgium

ARTICLE INFO

Article history:
Received 29 August 2011
Received in revised form
22 November 2012
Accepted 11 December 2012
Available online 7 March 2013

Keywords:
Identification algorithms
Parameter estimation
Nonlinear systems
Nonlinear models
State-space models
Block-structured models

ABSTRACT

This paper presents a method that generates initial estimates for a rather general block-structured model, starting from the (more general) polynomial nonlinear state-space model. The considered block-structure, sometimes referred to as Linear Fractional Transformation (LFT) or Linear Fractional Representation (LFR), encompasses several simpler structures. It can e.g. describe Wiener, Hammerstein, Wiener–Hammerstein and nonlinear feedback structures. In fact, the chosen block-structure is the most general representation of a system with one Single-Input Single-Output (SISO) static nonlinearity. As is quite common in block-structure identification, the states and internal signals are assumed to be unknown. The method gradually imposes the structure of the LFR system, and at the same time finds an estimate of the Multiple-Input Multiple-Output (MIMO) linear dynamic part and the static nonlinearity (SNL). The method is illustrated via an experimental-data example.

1. Introduction

1.1. Goal, model description and related literature

The aim of this paper is to generate initial estimates for the identification of the block-structured model shown in Fig. 1 (which will be termed "Poly-LFR" from now since the static nonlinearity is assumed to be polynomial), from input–output data (a Single-Input Single-Output (SISO) system is assumed for simplicity). The model consists of a MIMO linear dynamic block (which can be decomposed in four SISO linear dynamic blocks) and one static nonlinearity (SNL). The attractive aspect of this structure is its possibility to represent several other nonlinear block-structures, such as Wiener, Hammerstein, Wiener–Hammerstein and nonlinear feedback structures. In fact, it is the most general representation of a system with a single SISO SNL.

An identification method for the Poly-LFR model structure is explained in Vandersteen and Schoukens (1999), but is based on time-consuming two-tone excitations and can only handle nonlinear degrees up to 3. The method to be presented can deal with more general excitation signals and has no limitations with respect to the nonlinear degree. The LFR model structure also appears – in a different context – in e.g. Ishido, Takaba, and Quevedo (2011) and Novara, Vincent, Hsu, Milanese, and Poolla (2011) and the references herein.

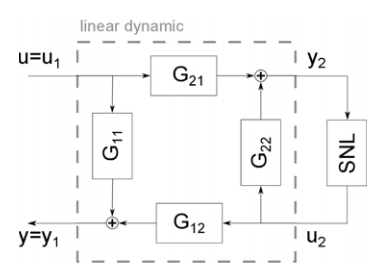

Fig. 1. Poly-LFR block structure with one static nonlinearity. Herein, the G_{ij} represent linear dynamic blocks.

1.2. Identification of the model

This paper proposes a two-step method to generate initial estimates of the Poly-LFR model: (i) a low-rank version of the Polynomial Nonlinear State-Space (PNLSS) model (Paduart et al., 2010) is estimated; (ii) the necessary conditions are imposed on it. Since the low-rank PNLSS is more general than the Poly-LFR,

☆ The material in this paper was partially presented at the 16th IFAC Symposium on System Identification (SYSID 2012), July 11–13, 2012, Brussels, Belgium. This paper was recommended for publication in revised form by Associate Editor Brett Ninness under the direction of Editor Torsten Söderström.

E-mail addresses: Anne.Van.Mulders@vub.ac.be (A. Van Mulders), Johan.Schoukens@vub.ac.be (J. Schoukens), Laurent.Vanbeylen@vub.ac.be (L. Vanbeylen).

[1] Tel.: +32 2 629 29 79; fax: +32 2 629 28 50.

0005-1098/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2013.01.052

the method can be regarded as an overparameterisation approach. Directly identifying the Poly-LFR from a linear initialisation leads to an increased risk of local minima compared to the proposed method.

1.3. Paper outline

In Section 2, assumptions on the model structure and other properties of the method are discussed. Section 3 presents respectively the PNLSS model structure and the target block-structured model, together with the interconnection between both. Section 4 explains how to obtain a low-rank description of the nonlinear parameter matrices. Section 5 discusses how to analytically determine the linear combination of states and input(s) that can be used as input of the SNL, as well as the computation of the SNL's parameters. The method is summarised in Section 6. An experimental-data example and conclusions can be found in respectively Sections 7 and 8.

2. Properties of the method

The proposed method avoids two common difficulties of the identification of block-structures: the identification of the separate subblock model orders and their parametric initialisation. In our approach, the following four main assumptions are made:

- there is only one SNL in the system;
- the direct-feedthrough matrix of G_{22} is zero, i.e. there is no direct term in the feedback branch;
- the SNL is of polynomial kind;
- the input signal used for identification is persistently exciting[2] the system.

The method does not need any special kind of excitation signal, or explicit knowledge of the model orders of the four linear blocks.

3. PNLSS model and target (Poly-LFR) model

Discrete-time state-space models are used throughout this paper. The states are assumed to be unknown and the final model should minimise the least-square error between the measured and modelled output.

Both the PNLSS model and the Poly-LFR model have structural degenerations that yield the same input–output behaviour. However, this is not an issue in this paper, since the only goal of the identification is to find a model that has an input–output behaviour similar to the true system.

3.1. PNLSS model structure

As is very well known, state-space models are well suited for multiple-input multiple-output (MIMO) systems. The PNLSS model is a conventional linear state-space model extended with polynomial terms $E\zeta$ and $F\zeta$:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + E\zeta(x(t), u(t)) \\ y_\theta(t) &= Cx(t) + Du(t) + F\zeta(x(t), u(t)) \\ y(t) &= y_\theta(t) + e(t) \end{aligned} \quad (1)$$

$x(t) \in \mathbb{R}^n$ are the states and $u(t) \in \mathbb{R}^{n_u}$ and $y(t) \in \mathbb{R}^{n_y}$ are the input and output. n is the model order, n_u is the number of inputs and n_y is the number of outputs. The top and bottom equations are called state and output equation. The input $u(t)$ is assumed to be known exactly and the output measurements $y(t)$ are corrupted via the noise term $e(t) \in \mathbb{R}^{n_y}$ (with coloured Gaussian noise with zero mean and finite variance). Under these assumptions, the weighted least-squares estimator corresponds to the maximum-likelihood estimator, which is asymptotically consistent, efficient and normally distributed (Kendall & Stuart, 1979). The vector $\zeta \in \mathbb{R}^{n_\zeta}$ contains monomials in $x(t)$ and $u(t)$; the matrices $E \in \mathbb{R}^{n\times n_\zeta}$ and $F \in \mathbb{R}^{n_y\times n_\zeta}$ contain the coefficients associated with those monomials. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n_u}$, $C \in \mathbb{R}^{n_y\times n}$ and $D \in \mathbb{R}^{n_y\times n_u}$ are constant coefficient matrices. The parameter vector is defined as

$$\theta^T = \left(vec(A)^T\; vec(B)^T\; vec(C)^T vec(D)^T\; vec(E)^T\; vec(F)^T\right). \quad (2)$$

From now on, although the method can easily be extended, only SISO systems ($n_u = n_y = 1$) are considered.

An identification method for this model is described in Paduart et al. (2010).

3.2. The target (Poly-LFR) block-structure

In this section, the equations of the Poly-LFR model are shown in their PNLSS form. First consider the state-space representation of the MIMO linear part, in which an explicit distinction between the contributions from $u = u_1, u_2$ to $y_\psi = y_1$ and y_2 is made:

$$\begin{aligned} x(t+1) &= Ax(t) + \begin{pmatrix} B & B_{NL}\end{pmatrix}\begin{pmatrix} u(t) \\ u_2(t)\end{pmatrix} \\ \begin{pmatrix} y_\psi(t) \\ y_2(t)\end{pmatrix} &= \begin{pmatrix} C \\ C_V\end{pmatrix} x(t) + \begin{pmatrix} D & D_{NL} \\ D_V & D_{22}=0\end{pmatrix}\begin{pmatrix} u(t) \\ u_2(t)\end{pmatrix} \\ y(t) &= y_\psi(t) + e(t) \end{aligned} \quad (3)$$

with

$$u_2(t) = \sum_{p=1}^{d} \alpha_p y_2^p(t)$$

and ψ the parameter vector of the Poly-LFR model. The nonlinear part determines the static polynomial relation between y_2 and u_2.

Rewriting the state- and true output equations yields

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + B_{NL}u_2(t) \\ y_\psi(t) &= Cx(t) + Du(t) + D_{NL}u_2(t) \end{aligned} \quad (4)$$

which, knowing that

$$u_2(t) = \sum_{p=1}^{d} \alpha_p \left(C_V x(t) + D_V u(t)\right)^p \quad (5)$$

is a PNLSS model (1). In the following, without loss of generality, A, B, C and D in (4) are assumed to be redefined such that $\alpha_1 = 0$ in (5), and the representation fits better with (1), in which monomials of degree 2 to d are considered.

For convenience, expression (5) is rewritten in terms of the monomials as

$$u_2(t) = V_1^T \zeta\,(x(t), u(t)). \quad (6)$$

Equating (4) and (6) with (1) results in

$$\begin{pmatrix} E \\ F\end{pmatrix} = \begin{pmatrix} B_{NL} \\ D_{NL}\end{pmatrix} V_1^T.$$

The nonlinear terms are restricted in two ways:

(1) $(E; F)$ is a rank-1 matrix: $(E; F) = U_1 V_1^T$, with $U_1 = (B_{NL}; D_{NL})$ a single column and V_1^T a single row.

[2] The input signal u_* is said to be persistently exciting at θ_* if

$$y_\theta(u_*) = y_{\theta_*}(u_*) \Rightarrow M(\theta_*) = M(\theta)$$

with $y_\theta(u)$ the response of the system to input signal u and manifold $M(\theta) = \left\{\theta_1 | \forall u, y_{\theta_1}(u) = y_\theta(u)\right\}$ defining the degeneracies of the model.