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Brief paper

Adaptive control for planar curve tracking under controller uncertainty[☆]

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ABSTRACT

Curve tracking is a challenging and central problem in robotics. In this note, we study adaptive control and parameter estimation for two dimensional robotic curve tracking with unknown control gains. We build adaptive controllers that stabilize equilibria corresponding to a fixed constant distance to the curve and zero bearing. The significance of our work is in (a) our ability to identify the unknown control gain, (b) our proof of input-to-state stability with respect to actuator errors, and (c) the optimality of our disturbance bounds for maintaining robust forward invariance of a nested class of hexagons that fill the state space, under actuator disturbances and the unknown control gain. We demonstrate our work in simulations.

1. Introduction

Curve tracking is of fundamental importance for the effective navigation of mobile robots in complex environments (Lumelsky & Stepanov, 1987; Woolsey & Techy, 2009). For wheeled mobile robots, feedback control laws based on Frenet–Serret frames have been developed that achieve autonomous tracking of obstacle boundaries or a desired smooth path (Micaelli & Samson, 1993). There were significant efforts to generalize the control law for mobile robots to adaptive path following control for under-actuated autonomous ships (Aguiar & Hespanha, 2007; Borhaug & Pettersen, 2005; Do, Jiang, & Pan, 2004). Several recent improvements for curve tracking control are reviewed in Morin and Samson (2008). These include extensions to the three dimensional case (Justh & Krishnaprasad, 2005) and cooperative control for ocean sensing (Zhang, Fratantoni, Paley, Lund, & Leonard, 2007; Zhang & Leonard, 2007).

Experimental evidence of robustness has been reported for farming (Lenain, Thuilot, Cariou, & Martinet, 2006), obstacle avoidance in corridors (Zhang, O'Connor, Luebke, & Krishnaprasad, 2004), ship control (Do & Pan, 2006) and ocean sampling (Zhang et al., 2007). Our previous work (Malisoff, Mazenc, & Zhang, 2012) theoretically justified the robustness by showing that the feedback controls in Zhang, Justh, and Krishnaprasad (2004) provide input-to-state stability (ISS) with respect to additive uncertainty on the controller; see Sontag (2006) for background on ISS.

Given a closed bounded subset H of the workspace, a key problem is to find the largest possible constant $\delta_H > 0$ such that all trajectories starting in H for all uncertainties bounded by δ_H remain in H at all future times. This is the problem of rendering H robustly forwardly invariant. While difficult to solve in general, our work (Malisoff et al., 2012) solved this problem for a nested sequence $\{H_i\}$ of forward invariant hexagons that fill the workspace for the two dimensional curve tracking dynamics from Zhang, Justh et al. (2004). By combining ISS with robustly forward invariant sets, we gave predictable tolerance and safety bounds for the curve tracking control laws.

This paper continues our search (begun in Malisoff et al., 2012; Zhang, Justh et al., 2004) for robust controller designs for curve tracking dynamics that respect the constraints in real time implementations. We are motivated by our recent deployment of marine robots at Grand Isle State Park, Louisiana which searched for residual crude oil components from the Deepwater Horizon oil spill disaster (Zhang, 2011). In this application, we implemented the curve tracking controllers from Malisoff et al. (2012) and Zhang, Justh et al. (2004) on two marine robots with unknown actuator parameters by testing different control gains. However,

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one significant limitation in Malisoff et al. (2012) and Zhang, Justh et al. (2004) is that they assume that the control gains are known, which can be a very demanding condition. Clearly, the existing results would be more applicable to real systems if they were adaptive to unknown actuation parameters.

In this paper, we present an adaptive controller that can be implemented on marine robots to track two-dimensional curves. The significance of our work is in (a) our ability to identify the unknown control gain caused by unknown actuation parameters, (b) the proof of ISS with respect to actuator errors, and (c) the maximality of the disturbance bounds for maintaining forward invariance of a class of hexagons under actuator disturbances and the unknown control gain. In addition, our proof of ISS and explicit estimates on the disturbance bounds have not been seen in the literature. The works (Aguiar & Hespanha, 2007; Do et al., 2004) handle more complex ship models than ours, without proving ISS. Nevertheless, by identifying the parameters and proving ISS and robust forward invariance, our results mark a significant theoretical step for curve tracking that lends itself to the applications we encountered in field work.

2. Notation and definitions

We use the standard classes of comparison functions $\mathcal{K}_\infty$ and $\mathcal{KL}$ (Sontag, 2006). Consider any subset $\mathcal{G}$ of a Euclidean space and any point $\mathcal{E} \in \mathcal{G}$. We use the usual definitions of positive definiteness, negative definiteness, positive semidefiniteness, and negative semidefiniteness with respect to $\mathcal{E}$, and moduli and nonstrict and strict Lyapunov functions with respect to $(\mathcal{E}, \mathcal{G})$; see Malisoff et al. (2012) for their standard statements. Let $|p|_\mathcal{E} = |p - \mathcal{E}|$ denote the distance between any point $p \in \mathcal{G}$ and $\mathcal{E}$, in the usual Euclidean metric. For any subset $\mathcal{H} \subseteq \mathbb{R}^n$ and any point $\bar{p} \in \mathbb{R}^n$, we set $\mathcal{H} - \bar{p} = \{q - \bar{p} : q \in \mathcal{H}\}$.

Let $\mathcal{U}$ be any subset of a Euclidean space such that $0 \in \mathcal{U}$. Let $|f|_\mathcal{S}$ denote the essential supremum of any function f over any set $\mathcal{S}$, and $|f|_\infty$ denote its essential supremum over its entire domain. Take any forward complete system

$$\dot{x} = \mathcal{F}(x, \delta) \tag{1}$$

with state space $\mathcal{G}$ and measurable essentially bounded disturbances $\delta : [0, +\infty) \to \mathcal{U}$, where $\mathcal{F} : \mathcal{G} \times \mathcal{U} \to \mathcal{G}$ satisfies the standard existence and uniqueness of solutions properties and $\mathcal{F}(\mathcal{E}, 0) = 0$. Let $\mathcal{S} \subseteq \mathcal{G}$ be any neighborhood of $\mathcal{E}$. We say that the system is *input-to-state stable (ISS)* with respect to $(\mathcal{U}, \mathcal{E}, \mathcal{S})$ provided there are functions $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}_\infty$ and a modulus Λ with respect to $\mathcal{S}$ such that

$$|x(t, x_0, \delta)|_\mathcal{E} \le \beta(\Lambda(x_0), t) + \gamma(|\delta|_{[0,t]}) \tag{2}$$

for all $t \ge 0$, all solutions $x(t, x_0, \delta)$ of the system with initial states $x_0 \in \mathcal{S}$, and all disturbances δ valued in $\mathcal{U}$. This agrees with the usual ISS condition when $\mathcal{G} = \mathcal{S} = \mathbb{R}^n$, $\mathcal{E} = 0$, and $\Lambda(x) = |x|$. The special case of ISS where $\mathcal{F}$ only depends on x and the term $\gamma(|\delta|_{[0,t]})$ in (2) is not present is *global asymptotic stability (GAS)* with respect to $(\mathcal{E}, \mathcal{S})$. A set $\mathcal{H} \subseteq \mathcal{G}$ is *robustly forwardly invariant for* (1) *with disturbances in* $\mathcal{U}$ provided all trajectories of (1) with initial states in $\mathcal{H}$ and disturbances δ valued in $\mathcal{U}$ remain in $\mathcal{H}$ for all positive times.

3. Problem formulation

The curve tracking dynamics can be simplified to

$$\dot{\rho} = -\sin(\phi), \qquad \dot{\phi} = \frac{\kappa\cos(\phi)}{1 + \kappa\rho} - K(u + \delta) \tag{3}$$

where ρ is the distance between the robot and the curve being tracked, ϕ is the bearing, κ is the positive valued curvature at the closest point on the curve, u is the steering control, the real valued essentially bounded perturbation δ represents controller uncertainty, and the state space is $\mathcal{X} = (0, +\infty) \times (-\pi/2, \pi/2)$ (Zhang, Justh et al., 2004). The control gain $K > 0$ is constant (and possibly uncertain). The dynamics (3) are obtained by forming the Frenet–Serret systems for the robot and the curve being tracked and taking ϕ to be the angle between the tangent vectors for the robot trajectory and the curve it is tracking (Zhang, Justh et al., 2004). While (3) is feedback linearizable, the feedback linearized system does not respect the state space and so cannot be used to prove ISS (Malisoff et al., 2012).

When K is known and κ is constant, Zhang, Justh et al. (2004) designed a feedback law to achieve asymptotic stabilization of an equilibrium corresponding to a constant distance ($\rho = \rho_0 > 0$) and zero bearing ($\phi = 0$), which occurs when the robot moves parallel to the curve. This is the nonadaptive case. Here, our goals for an adaptive control law are to have K identified in addition to achieving $\rho \to \rho_0$ and $\phi \to 0$ as $t \to +\infty$, when $\delta = 0$. Then, we would like to establish ISS properties for the closed loop dynamics under the adaptive control. This is more challenging than what has been addressed in previous works, which have the serious limitations of assuming that K is known or not proving ISS.

We solve this adaptive stabilization and parameter identification problem using a significant extension of Mazenc, Malisoff, and de Queiroz (2011). Three shortcomings of Mazenc et al. (2011) are that (a) its controllers do not apply under state constraints such as our set $\mathcal{X}$, (b) they do not lead to the key ISS property with respect to controller uncertainty, and (c) they cannot provide crucial robust forward invariance properties that ensure that the robot maintains a safe configuration by respecting a nested sequence of state constraints. Therefore, in addition to overcoming the shortcomings of Malisoff et al. (2012) for our curve tracking model, we provide a significant improvement of the nonadaptive invariant hexagon argument from Malisoff et al. (2012) that ensures robust forward invariance of suitable hexagons for the adaptive dynamics, under maximal bounds on the perturbation δ.

4. Review of a nonadaptive case

To make our paper self-contained, we review the relevant results from Malisoff et al. (2012); Zhang, Justh et al. (2004), which use controllers of the form

$$u_0 = \frac{\kappa\cos(\phi)}{1 + \kappa\rho} - h'(\rho)\cos(\phi) + \mu\sin(\phi) \tag{4}$$

and constant positive curvatures, where $\mu > 0$ is a steering constant. In Malisoff et al. (2012), h is any function that satisfies these two assumptions:

Assumption 1. $h : (0, +\infty) \to [0, +\infty)$ is C^2, h' has only finitely many zeros, $\lim_{\rho\to 0^+} h(\rho) = \lim_{\rho\to+\infty} h(\rho) = +\infty$, and there is a constant $\rho_0 > 0$ such that $h(\rho_0) = 0$.

Assumption 2. Assumption 1 holds and:

(a) There is an increasing C^1 function $\gamma : [0, +\infty) \to [\mu, +\infty)$ such that for all $\rho > 0$, we have $\gamma(h(\rho)) \ge 1 + 0.5\mu^2 + h''(\rho)$.
(b) There is a function $\Gamma \in \mathcal{K}_\infty \cap C^1$ such that $\Gamma(h(\rho)) \ge [h'(\rho)]^2$ for all $\rho > 0$.
(c) $h'(\rho)(\rho - \rho_0)$ is positive for all $\rho > 0$ except $\rho = \rho_0$.

Remark 1. Positivity of ρ precludes bringing the robot exactly on the curve. The arguments in Malisoff et al. (2012) show that Assumptions 1–2 hold for

$$h(\rho) = \alpha\left(\rho + \frac{\rho_0^2}{\rho} - 2\rho_0\right) \tag{5}$$