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Recursive estimation for nonlinear stochastic systems with multi-step transmission delays, multiple packet dropouts and correlated noises

Shaoying Wang [a,b], Huajing Fang [a,*], Xuegang Tian [b]

[a] School of Automation, Huazhong University of Science and Technology, Wuhan 430074, China
[b] Department of Mathematics, Binzhou University, Shandong 256603, China

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ABSTRACT

This paper is concerned with the recursive estimation problem for a class of discrete-time nonlinear stochastic systems with multi-step transmission delays, multiple packet dropouts and correlated noises. The stochastic nonlinearity is described by statistical means, and noises are assumed to be one-step autocorrelated and cross-correlated. To convert the original system into the nonlinear stochastic parameterized one, some new variables are firstly introduced. Then, by applying the innovation analysis approach, the optimal linear estimators including filter, multi-step predictor and smoother are presented. The proposed algorithms, which are dependent on the probabilities of delays and data losses, the matrices used to describe the stochastic nonlinearity as well as one-step correlation coefficient matrices, are expressed by the Riccati and Lyapunov equations. Furthermore, sufficient conditions are established to guarantee the convergence of the state covariance and the existence of the steady-state estimators for the time-invariant nonlinear systems. Finally, a simulation example is given to demonstrate the effectiveness of the proposed algorithms.

1. Introduction

In recent years, the state estimation problem for networked systems has attracted considerable attention because of its wide application in many areas, such as communication, navigation, target tracking, computer vision, fault diagnosis and so on [1,2]. With the insertion of communication networks, packet dropouts, transmission delays and missing measurements may occur during data transmission. Thus the data received by the estimator may not be real time ones, which lead to the traditional estimation algorithms such as the well-known Kalman filtering being no longer applicable. Therefore, it is significant to investigate new estimation algorithms for networked systems with imperfect measurements.

So far, a great number of results regarding state estimation have been reported for the linear networked systems. For the case of transmission delays, [3,4] proposed the measurement reorganization approach to deal with the filtering problems, where the delayed system was transformed into a delay-free one. By means of linear matrix inequalities, the event-based H_∞ filtering problem was investigated in [5]. For the system with multiple packet dropouts, the optimal linear estimators were designed in [6] based on the innovation analysis approach. In [7], Kalman filtering with intermittent observations was analyzed. Furthermore, based on the projection theorem [8], the robust Kalman filtering problem with random sensor delays and multiple packet dropouts was investigated in [9]. When random measurement delays, packet dropouts and missing measurements exist simultaneously in the system, the optimal H_2 filtering [10], the adaptive Kalman filtering [11], the optimal linear estimator design algorithms [12] and recursive least-squares linear estimation algorithms

* Corresponding author.
E-mail address: hjfang@mail.hust.edu.cn (H. Fang).

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[13] were respectively proposed. In particular, the results in [13] were obtained only employing covariance information, without resorting to state space model. It should be note that, in most cases, the delay model is assumed to be one-step. Recently, [14] developed a novel model to describe multi-step transmission delays and multiple packet dropouts existing both from the sensors to the controller and from the controller to the actuators. And the optimal linear estimators including predictor, filter and smoother were given.

However, the noises involved in the aforementioned references are uncorrelated with each other or cross-correlated at the same time instant. Such assumption is not always realistic in practical applications [15–18]. On the other hand, owing to sudden environment changes, random failures and repairs of the components, stochastic nonlinearity inevitably exists in many engineering systems, and may cause these systems perform poorly [1]. Accordingly, great research interests have been focused on these topics and a large number of results have been obtained, see, e.g., [19–30]. More specifically, [18] gave the optimal linear estimator and analyzed its limiting behavior for the system with stochastic nonlinearities. Ref. [20] investigated the robust filter design problem for nonlinear systems with multiplicative noises. For a class of nonlinear systems with multiple missing measurements, [21] discussed the robust variance-constrained filtering problem and derived a sufficient condition for the exponential mean-square stability of the filtering error. Taking random parameter matrices, multiple fading measurements and correlated noises into account, [24] presented an unbiased recursive filter, where the process noise and the measurement noise are one-step autocorrelated and two-step cross-correlated. Assuming the process noise to be finite-step autocorrelated, [28] developed the gain-constrained recursive filtering algorithm for the nonlinear stochastic systems with probabilistic sensor delays. Unfortunately, the above references only design the filter, not refer to the multi-step predictor and smoother. Moreover, the issue of the steady-state estimator is rarely involved.

Motivated by the above discussion, this paper focuses on the recursive estimation for the nonlinear stochastic systems with bounded transmission delays, multiple packet dropouts and one-step autocorrelated and cross-correlated noises. In [7,12,14], the innovation analysis approach has been proved to be an effective tool to design filter, multi-step predictor and smoother for linear systems. In the current paper, the stochastic nonlinearity characterized by the statistical means enter into the system model. Considering the statistical properties of the stochastic nonlinearity, the approach employed in [7,12,14] is also suitable to propose the optimal estimators for stochastic nonlinear system [19]. Meanwhile, unlike the one-step delay model [13], 2-step random delays and packet dropout model is introduced in this paper. And the process and measurement noises are supposed to be autocorrelated and cross-correlated. A set of new variables is defined to transform the original nonlinear system into the nonlinear stochastic parameterized one, from which the optimal estimators are presented. Moreover, the steady-state estimators are discussed for the time-invariant nonlinear system. The main contributions of this paper can be summarized as follows: (1) the considered system model is comprehensive that covers stochastic nonlinearity, transmission delays, packet dropouts as well as correlated noises, (2) using the orthogonal projection theory, the optimal filter, multi-step predictor and smoother are proposed for the nonlinear stochastic system, (3) with resort to the stacking operator and the Kronecker product of matrices, the asymptotical stability of the steady-state estimators for the time-invariant nonlinear system is discussed, (4) the proposed estimation algorithms are expressed by the recursion equations which are suitable for online computation.

The rest of this paper is organized as follows. Section 2 states the problem under investigation. In Section 3, the derivation of the optimal linear filter, predictor and smoother is presented based on the innovation analysis approach. In Section 4, sufficient conditions for the convergence of the state covariance and the existence of the steady-state estimators are obtained. Finally, a numerical example is given in Section 5, which is followed by some conclusions in Section 6.

Notation: Throughout this paper, R^n denotes the n-dimensional Euclidean space. $E(x)$ stands for the expectation of x. The notion $X>Y$ means that $X-Y$ is positive definite, where X and Y are symmetric matrices. $\rho(A)$ represents the spectral radius of matrix A. st(A) means the stack form of the matrix A, while tr(A) is the trace of the matrix A. The superscript T stands for matrix transpose. $\otimes$ denotes the Kronecker product of matrices. $\delta_{t,l}$ is the Kronecker delta function, which is equal to one, if $t=l$, and zero otherwise. $*$ represents the same elements as in the preceding brace $\{\}$.

2. Problem formulation

Consider the following discrete time-varying nonlinear stochastic system:

$$x(t+1)=A(t)x(t)+f(x(t),\xi(t))+B(t)w(t) \tag{1}$$

$$z(t)=C(t)x(t)+v(t) \tag{2}$$

where $x(t)\in R^n$ is the state vector, $z(t)\in R^m$ is the measurement output. $w(t)\in R^h$ and $v(t)\in R^m$ are zero mean process and measurement noises, respectively. $A(t), B(t)$ and $C(t)$ are time-varying matrices with appropriate dimensions. $f(x(t),\xi(t))$ represents the stochastic nonlinearity of the state. $\xi(t)$ is a zero mean Gaussian noise sequence that is uncorrelated with other noise signals.

As depicted in Fig. 1, the measurement $z(t)$, transmitted to the estimator through a communication network, may be random delayed and/or lost. Inspired by the multi-step delays and packet dropouts model in [14], we will adopt 2-step delays model in the current paper, which makes our theory become more understandable and acceptable because the case of 2-step delays not only can reveal the fundamental principles of our proposed estimators, but also can simplify their derivation procedures. So $y(t)$ is given as follows:

$$\begin{aligned}y(t)=&r_0(t)z(t)+(1-r_0(t))[(1-r_0(t-1))r_1(t)z(t-1)\\&+[1-(1-r_0(t-1))r_1(t)](1-r_0(t-2))(1-r_1(t-1))r_2(t)z(t-2)]\end{aligned} \tag{3}$$

where $r_i(t)$ $(i=0,1,2)$ are Bernoulli distributed random variables satisfying $\text{Prob}\{r_i(t)=1\}=\alpha_i$ and $\text{Prob}\{r_i(t)=0\}=$