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Fast communication

Weighted least squares algorithm for target localization in distributed MIMO radar

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ABSTRACT

In this paper, we address the problem of locating a target using multiple-input multiple-output (MIMO) radar with widely separated antennas. Through linearizing the bistatic range measurements, which correspond to the sum of transmitter-to-target and target-to-receiver distances, a quadratically constrained quadratic program (QCQP) for target localization is formulated. The solution of the QCQP is proved to be an unbiased position estimate whose variance equals the Cramér–Rao lower bound. A weighted least squares algorithm is developed to realize the QCQP. Simulation results are included to demonstrate the high accuracy of the proposed MIMO radar positioning approach.

1. Introduction

Target localization [1] has been one of the central problems in many fields such as radar [2], sonar [3], telecommunications [4], mobile communications [5], sensor networks [6] as well as human–computer interaction [7]. Recently, this topic has also received considerable interest in multiple-input multiple-output (MIMO) radar [8–13]. Unlike the conventional phased-array radar which deals with a single waveform, MIMO radar employs multiple antennas to transmit and receive different waveforms and process the received signals. The MIMO radar with colocated antennas utilizes waveform diversity while that with widely separated antennas provides spatial diversity, and both of them are superior to its phased-array counterpart in many aspects. In this work, we address target localization using the latter architecture.

Generally speaking, MIMO radar positioning with distributed antennas can be classified as direct and indirect approaches. In the former, the target location is estimated via processing the observed receiver outputs directly, and representative examples include the maximum likelihood (ML) [9,10] and sparse recovery [11] methods. On the other hand, there are two steps in the indirect approach. First, the time delay information, which corresponds to the sum of the signal propagation time from a transmit antenna to the target and that from the target to a receive antenna, is estimated from the received data. In the second step, we multiply these delays by the known signal propagation speed to yield the bistatic range estimates for constructing a set of elliptic equations from which the source position can be determined. Godrich et al. [9] have proposed a best linear unbiased estimator (BLUE) by linearizing the elliptic equations via Taylor series expansion [14] while a different linearization scheme is devised in [12], which results in solving two sets of linear equations. We refer the latter solution to as the combined linear least squares (CLLS) method. Although the BLUE is superior to the CLLS solution

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and can attain ML performance, an initial position estimate which is sufficiently close to the source location is required. Generally speaking, the direct methods are more computationally demanding than the indirect approach. For example, according to [9], when the ML estimator is realized by a grid search, a significant computational effort is involved. The complexity of the sparse modeling approach [11] is also high because convex optimization is involved. Inspired by the linearization approach of Chan and Ho [15], we devise an accurate indirect positioning approach for distributed MIMO radar where no algorithm initialization is needed. Our contributions can be summarized as (i) the problem of solving the elliptic equations constructed from the bistatic range estimates is converted to a quadratically constrained quadratic program (QCQP); (ii) we have proved that under sufficiently small noise conditions, the solution of the QCQP is an unbiased estimate of the target location and its variance is equal to the optimality benchmark, namely, the Cramér–Rao lower bound (CRLB); and (iii) analogous to the relaxation technique in [15], we develop a weighted least squares (WLS) estimator for solving the QCQP.

The rest of the paper is organized as follows. In Section 2, the problem of distributed MIMO radar positioning is formulated. In Section 3, we first convert the target localization formulation to a QCQP. Bias and mean square error (MSE) of the estimate of the QCQP are also analyzed. The WLS algorithm for realizing the QCQP is then developed. Simulation results are presented in Section 4 to evaluate the localization accuracy of the proposed approach by comparing with the CLLS method and the CRLB. Finally, conclusions are drawn in Section 5.

The notation is introduced as follows. Scalars, vectors and matrices are denoted by italic, bold lower-case and bold upper-case symbols, respectively. An estimate of a is denoted by $\hat{a}$. The ith element of $\mathbf{a}$ and (i,j) entry of $\mathbf{A}$ are represented as $[\mathbf{a}]_i$ and $[\mathbf{A}]_{i,j}$, respectively. Moreover, $[\mathbf{A}]_{i:j,k:l}$ contains entries in the intersection of the ith to the jth rows and the kth to the lth columns. The Euclidian norm of $\mathbf{a}$ is denoted by $\|\mathbf{a}\|_2$. The ^{T} and $^{-1}$ denote the matrix transpose and inverse, respectively, while E is the expectation operator. The $\mathrm{diag}(a_1, a_2, \ldots, a_k)$ is a diagonal matrix with diagonal elements $a_1, a_2, \ldots, a_k$. The $\mathbf{0}_{i\times j}$ and $\mathbf{I}_i$ represent the $i\times j$ zero matrix and $i\times i$ identity matrix, respectively.

2. Problem formulation

We consider finding the position of a target, denoted by $\mathbf{x} = [x\ y]^T$, using a distributed MIMO radar system with M transmit and N receive antennas whose coordinates, $\mathbf{x}_m^t = [x_m^t\ y_m^t]^T$, $m = 1, 2, \ldots, M$, and $\mathbf{x}_n^r = [x_n^r\ y_n^r]^T$, $n = 1, 2, \ldots, N$, respectively, are known. Note that although we assume two-dimensional positioning, extension to the three-dimensional case is straightforward. Suppose the transmit antennas send a set of orthogonal waveforms with center frequency f_c and bandwidth Δf. Let $s_m(t)$ be the low-pass equivalent of the emitted signal from the mth transmitter. The transmitted waveforms are reflected by the target and then collected at the receive antennas. In the case of coherent processing where the receivers are phase-synchronized, the signal measured at the nth antenna, denoted by $z_n(t)$, is [9,12]

$$z_n(t) = \sum_{m=1}^{M} \alpha_{m,n} \exp\{-j2\pi f_c \tau_{m,n}\} s_m(t-\tau_{m,n}) + w_n(t). \tag{1}$$

Here, $\alpha_{m,n}$ is the amplitude corresponding to the path from the mth transmitter to the nth receiver, $\tau_{m,n}$ is the sum of the signal propagation time from the mth transmitter to the target and the time from the target to the nth receiver, denoted by τ_m^t and τ_n^r, respectively, and $w_n(t)$ is the zero-mean complex Gaussian noise which is temporally and spatially white with variance σ_w^2. That is, $\tau_{m,n}$ can be decomposed as

$$\tau_{m,n} = \tau_m^t + \tau_n^r, \quad m = 1, 2, \ldots, M, \quad n = 1, 2, \ldots, N \tag{2}$$

Denoting the wave propagation speed by c, τ_m^t and τ_n^r can also be expressed as

$$\tau_m^t = \frac{1}{c}\|\mathbf{x}_m^t - \mathbf{x}\|_2 = \frac{1}{c}\sqrt{(x_m^t - x)^2 + (y_m^t - y)^2} \tag{3}$$

$$\tau_n^r = \frac{1}{c}\|\mathbf{x}_n^r - \mathbf{x}\|_2 = \frac{1}{c}\sqrt{(x_n^r - x)^2 + (y_n^r - y)^2}. \tag{4}$$

For non-coherent processing model where the receive antennas are not phase-synchronized, the observed signal becomes [9,12]

$$z_n(t) = \sum_{m=1}^{M} \alpha_{m,n} s_m(t-\tau_{m,n}) + w_n(t). \tag{5}$$

In the direct approach, $\mathbf{x}$ is determined directly from (1) or (5). In this work, our focus is on the indirect approach where we assume that $\{\tau_{m,n}\}$ have been estimated from the received signals using the ML method. By multiplying the estimates by c, we obtain the bistatic range measurements, denoted by $\{r_{m,n}\}$, which are modeled as

$$r_{m,n} = R_m^t + R_n^r + \epsilon_{m,n}, \quad m = 1, 2, \ldots, M, \quad n = 1, 2, \ldots, N \tag{6}$$

where

$$R_m^t = \|\mathbf{x}_m^t - \mathbf{x}\|_2 \tag{7}$$

$$R_n^r = \|\mathbf{x}_n^r - \mathbf{x}\|_2. \tag{8}$$

Here, R_m^t and R_n^r are the unknown ranges between the target and the mth transmitter and the nth receiver, respectively, while $\epsilon_{m,n}$ is the zero-mean Gaussian distributed range estimation error, whose power is inversely proportional to $|\alpha_{m,n}|^2$. For presentation simplicity, we assume that $|\alpha_{m,n}| = |\alpha|$ for all m and n. In the coherent case, the covariance matrix of $\epsilon_{m,n}$, denoted by $\mathbf{C}_\epsilon$, is [9]

$$\mathbf{C}_\epsilon = \frac{c^2 \sigma_w^2}{8\pi^2 f_c^2 |\alpha|^2} \mathbf{I}_{MN}. \tag{9}$$

While for non-coherent processing, the corresponding covariance matrix is given by [9]

$$\mathbf{C}_\epsilon = \frac{c^2 \sigma_w^2}{8\pi^2 (\Delta f)^2 |\alpha|^2} \mathbf{I}_{MN}. \tag{10}$$

In both cases, $\mathbf{C}_\epsilon$ is a scaled identity matrix, indicating that $\epsilon_{m,n}$ is zero-mean white Gaussian process and we denote its variance as σ_ϵ^2. It is worth noting that our algorithm development and analysis can be straightforwardly extended to the case when each $\epsilon_{m,n}$ has distinct power, that is, $\mathbf{C}_\epsilon$ is a