Automatica 50 (2014) 421–430

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Control limitations from distributed sensing: Theory and Extremely Large Telescope application[☆]

Alain Sarlette[a,1], Rodolphe J. Sepulchre[b]

[a] SYSTeMS at Ghent University, Technologiepark Zwijnaarde 914, B-9052 Gent (Zwijnaarde), Belgium

[b] Control Group at the University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, United Kingdom

ARTICLE INFO

Article history:
Received 23 April 2013
Received in revised form
24 September 2013
Accepted 28 October 2013
Available online 2 January 2014

Keywords:
Distributed detection
Performance limitations
Robustness
Integral control
Telescopes
Distributed control

ABSTRACT

We investigate performance bounds for feedback control of distributed plants where the controller can be centralized (i.e. it has access to measurements from the whole plant), but sensors only measure differences between neighboring subsystem outputs. Such "distributed sensing" can be a technological necessity in applications where system size exceeds accuracy requirements by many orders of magnitude. We formulate how distributed sensing generally limits feedback performance robust to measurement noise and to model uncertainty, without assuming any controller restrictions (among others, no "distributed control" restriction). A major practical consequence is the necessity to cut down integral action on some modes. We particularize the results to spatially invariant systems and finally illustrate implications of our developments for stabilizing the segmented primary mirror of the European Extremely Large Telescope.

1. Introduction

The massive availability of sensors and actuators in our environment is an opportunity to address large-scale problems by exploiting their interaction. The control of interacting localized subsystems (distributed plants) has drawn tremendous interest in the last decades, covering e.g. agreement (consensus) in computer networks (Tsitsiklis, 1993), synchronization of dynamical systems (Nair & Leonard, 2008; Strogatz, 2003) or collective robotic task solving (Bullo, Cortés, & Martínez, 2009). A defining property is the information sharing between subsystems—information content, and interconnection topology. Common distinctions are reference-following (Lawton & Beard, 2002) vs. autonomous coordination (Strogatz, 2003) and centralized vs. distributed control (Bamieh, Jovanovic, Mitra, & Patterson, 2012; Bamieh, Paganini, & Dahleh, 2002; de Castro & Paganini, 2002; Gorinevsky, Boyd, & Stein, 2008; Langbort & D'Andrea, 2005; Stewart, Gorinevsky, & Dumont, 2003). In centralized control, each local action is a function of measurements all over the system. The distributed control paradigm (Bamieh et al., 2002; de Castro & Paganini, 2002; Gorinevsky et al., 2008; Langbort & D'Andrea, 2005; Stewart et al., 2003) imposes a localized coupling in closed-loop: each local action depends on neighboring subsystem outputs only.

This paper shows how, ahead of the controller choice, structural restrictions on the *sensing architecture* of a distributed plant can fundamentally constrain the performance of feedback. Specifically, we consider systems which only sense *differences between neighboring subsystem outputs*. Unlike in distributed control, we allow the resulting measurements to be used in any – in particular, centralized – control computation. We call this "local relative sensing" or "distributed sensing". It is motivated by applications in which communication capabilities allow to quickly broadcast all measurements and control signals – questioning a priori restrictions on controller structure – but sensor technology does not allow accurate enough absolute measurements over the entire plant. This occurs in multi-scale problems, where accuracy requirements and plant size differ by many orders of magnitude. The setting is inspired by our study of primary mirror stabilization for the European Extremely Large Telescope (EELT) (Bastin, Sarlette, & Sepulchre, 2009). We therefore propose indicative analytical results – for a general case and for 1-degree-of-freedom spatially invariant systems – followed by an illustration on this case study. We focus on two concerns. First, how distributed sensing influences the sensor noise vs. disturbance rejection tradeoff, using the sensitivity transfer functions of classical linear control theory (Åstrøm, 2000; Åstrøm & Murray, 2008). Second, how

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Peng Shi under the direction of Editor Toshiharu Sugie.

E-mail addresses: alain.sarlette@ugent.be (A. Sarlette), r.sepulchre@eng.cam.ac.uk (R.J. Sepulchre).

[1] Tel.: +32 9 264 5658; fax: +32 9 264 5840.

0005-1098/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.automatica.2013.12.014

measurement model errors affect robustness. A major concrete consequence is the necessity to cut down integral action on some modes.

Effects of noise and perturbations in distributed systems are examined in various ways in the literature. The authors of Bamieh et al. (2012), Barooah, Mehta, and Hespanha (2009) and Hao, Barooah, and Mehta (2011) restrict not only sensing, but also control, to local relative coupling (this is *distributed control*). The authors of Barooah and Hespanha (2007) study, in a static setting, bounds on the reconstruction of absolute position with respect to a leader from noisy local relative measurements. Optimal controllers for *spatially invariant* plants are investigated in Bamieh et al. (2002); for a locally coupled plant, the optimal gains decay exponentially as a function of distance between actuated and measured subsystems. This supports the use of distributed control, for which Bamieh et al. (2012) investigate performance limitations on a benchmark spatially invariant system. The robustness issue, that we first raised in Bastin et al. (2009), has been observed numerically with μ-analysis for the segmented mirror application (MacMynowski, 2009). Segmented mirror stabilization has been investigated by a few teams associated to Extremely Large Telescope projects (Jiang, Voulgaris, Holloway, & Thompson, 2009; MacMartin & Chanan, 2003; MacMynowski, 2009). In Jiang et al. (2009) the distributed sensing issue is put aside by assuming absolute measurements.

The paper is organized as follows. Section 2 formalizes distributed sensing and gives two motivating examples: a benchmark vehicle-chain problem and segmented mirror stabilization. Section 3 formulates how sensor noise (3.1) and model errors (3.2) induce performance limitations. Section 4 particularizes to 1-degree-of-freedom spatially invariant systems. Section 5 illustrates our point on EELT primary mirror stabilization.

Notation. We write $i = \sqrt{-1}$ the imaginary unit. The element on row j, column k of matrix $C \in \mathbb{C}^{l\times m}$ is denoted as $(C)_{j,k}$. C^T and C^* respectively denote transpose and complex conjugate transpose of C, and $\otimes$ the Kronecker product of two matrices. We denote $c \in \mathbb{C}^l$ a column vector, $\|c\| = \sqrt{\sum_k |c_k|^2}$ its Euclidean norm. $I_m \in \mathbb{R}^{m\times m}$ is the identity matrix and $\mathbf{1}_m \in \mathbb{R}^m$ the column-vector of all ones. We interpret $s+C = s\,I_m+C$ if $s \in \mathbb{C}$ and $C \in \mathbb{C}^{m\times m}$. For $D \in \mathbb{C}^{m\times m}$ diagonal and f a scalar function, $Y = f(D) \in \mathbb{C}^{m\times m}$ is diagonal with $(Y)_{k,k} = f((D)_{k,k})$ for all k.

2. Distributed sensing models

We consider a Laplace-domain model (see Fig. 1)

$$y(s) = G(s)\,[u(s) + d(s)] \tag{1}$$

$$z(s) = [B + \Delta]\,y(s) + n(s) \tag{2}$$

$$u(s) = -C(s)\,z(s) \tag{3}$$

to represent $M \gg 1$ coupled N-dimensional subsystems. Components $kN + 1$ to $(k + 1)N$ of $y(s), u(s), d(s) \in \mathbb{C}^{N_y}$ denote outputs, inputs and disturbances of subsystem k in the Laplace domain, with $N_y = NM$. We assume that the plant governed by $G(s)$ is stable. Output $z(s) \in \mathbb{C}^{N_z}$ is obtained through the static map $[B + \Delta] \in \mathbb{R}^{N_z\times N_y}$, where B is the nominal sensor behavior and Δ a sensor model error. Each sensor measurement is corrupted by zero-mean independent identical Gaussian white noise, represented by $n(s)$ with covariance matrix $\sigma^2 I_{N_z}$. For ease of presentation we assume $N_z \geq N_y$. The purpose of controller $C(s) \in \mathbb{C}^{N_y\times N_z}$ is to reject disturbances $d(s)$ from y. Importantly, we do not restrict the controller (3) to be distributed, i.e. we allow $C(s)$ to be a full matrix. We also allow the disturbances on different subsystems to

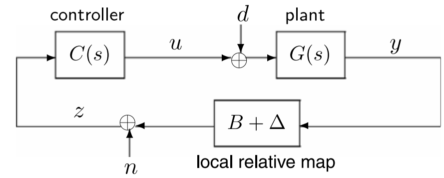

Fig. 1. Schematic representation of distributed sensing.

be correlated, by investigating how a general vector $d(s)$ affects the controlled plant. This differs from e.g. Bamieh et al. (2012) and Hao et al. (2011) which examine y for a given disturbance distribution (and controller).

The central element of our investigation is *local relative measurement*. Let $q_k = \{kN + 1, kN + 2, \ldots, (k + 1)N\}$.

Definition 1. B gives (unit-gain) *relative measurements* between subsystem outputs if for each $l \in \{1, \ldots, N_z\}$ there exist q_j, q_k such that

1. $(B)_{l,m} = 0 \quad$ for $m \notin q_j \cup q_k$
2. $$\sum_{m\in q_j} |(B)_{l,m}| = \sum_{m\in q_j} (B)_{l,m} = \sum_{m\in q_k} |(B)_{l,m}| = -\sum_{m\in q_k} (B)_{l,m} = 1.$$

That means, each row l of B measures the difference between a convex combination of outputs of subsystem j and a convex combination of outputs of subsystem k. For $N = 1$, B^T would be the *oriented incidence matrix* of some graph Γ_B, where subsystems are nodes and sensors are edges; for $N > 1$, B^T has the interpretation of a generalized incidence matrix, with matrix-valued weights on each edge (Barooah & Hespanha, 2007). The (generalized) Laplacian matrix of Γ_B is $L = B^TB$. If $(L)_{l,m} \neq 0$ for some $l \in q_j$ and $m \in q_k$, then subsystems j and k are connected in Γ_B.

Definition 2. A spatial structure $\mathcal{S}$ of dimension $\gamma \in \mathbb{N}$ associates a position $p(k) \in \mathbb{R}^\gamma$ to each subsystem k such that $\|p(k)-p(l)\| \geq 1$ for $l \neq k$.

Definition 3. Given a spatial structure and a fixed spatial range $\rho \geq 1$, measurement map B gives *local relative measurements of range* ρ if it gives relative measurements and it only connects in Γ_B subsystems for which $\|p(k) - p(l)\| \leq \rho$. We call this *distributed sensing*.

Many decentralized control settings associate a local measurement to each subsystem (graph node). With distributed sensing in contrast, *measurements are the result of interactions between subsystems (graph edges).*

Remark 1. Local sensing has no meaning if it is not relative. Sensors giving "absolute" e.g. positions actually physically measure positions with respect to a common ("central") reference physically shared among all sensors. Absolute measurements thus correspond to centralized sensing. This is also acknowledged in the robotics community, distinguishing local≅onboard from global≅offboard sensors, see e.g. Bräunl (2008, Chapter 3).

We study disturbance rejection limitations due to distributed sensing with $\frac{\rho}{M} \ll 1$. In the following two applications this arises as M increases with the size of a large-scale plant while ρ is limited by sensor technology.