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Robust stability and control of uncertain linear discrete-time periodic systems with time-delay[☆]

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ABSTRACT

This paper deals with the problems of robust stability analysis and robust control of linear discrete-time periodic systems with a delayed state and subject to polytopic-type parameter uncertainty in the state-space matrices. A robust stability criterion independent of the time-delay length as well as a delay-dependent criterion is proposed, where the former applies to the case of a constant time-delay and the latter allows for a time-varying delay lying in a given interval. The developed robust stability criteria are based on affinely uncertainty-dependent Lyapunov–Krasovskii functionals and are given in terms of linear matrix inequalities. These stability conditions are then applied to solve the problems of robust stabilization and robust H_∞ control via static periodic state feedback. Numerical examples illustrate the potentials of the proposed robust stability and control methods.

1. Introduction

Cyclic processes are quite often encountered in nature and engineering and thus periodic systems find application in a wide range of different fields, as for instance, economics, population dynamics, signal processing in the presence of cyclostationary noise, control of multirate plants, and multiplexed systems (see, for instance, Bittanti, 1986, Bittanti & Colaneri, 2009, and the references therein).

Over the past three decades linear periodic systems have been attracting significant interest within the control community and significant advances have been achieved in a variety of topics of the theories of control and state estimation. A large spectrum of important results on control and filtering analysis and synthesis have been developed to solve a variety of problems, as for instance, pole placement (Aeyels & Willems, 1995; Colaneri, 1991), characterization of all stabilizing controllers (Bittanti & Colaneri, 1999), robust stabilization (de Souza & Trofino, 2000; Farges, Peaucelle, Arzelier, & Daafouz, 2007), robust tracking (Grasselli, Longhi, Tornambè, & Valigi, 1996), H_∞ control (Colaneri & de Souza, 1992; Köse, 2002), H_2 control (Farges et al., 2007; Wiśniewskiand & Stoustrup, 2001), dissipativity (Yakubovich, Fradkov, Hill, & Proskurnikov, 2007), model predictive control (Gondhalekar & Jones, 2009), adaptive control (Tian & Narendra, 2009), adaptive robust regulation (Zhang & Serrani, 2009), fault detection (Fadali, Colaneri, & Nel, 2003), minimum mean-square state estimation (e.g., Bittanti, Colaneri, & De Nicolao, 1988, 1991 de Souza, 1991, and the references therein), and H_∞ filtering (Bittanti & Cuzzola, 2001; Xie & de Souza, 1993; Xie, de Souza, & Fragoso, 1991). In spite of all these developments, little attention has been devoted in the literature to the problems of robust stability analysis and robust control design for linear periodic systems with a time-delayed state. The motivation for considering time-delays in the framework of periodic systems is that time-delays are encountered in a number of applications due to transport of material lags and/or communication delays. A relevant example is in multirate control of networked control systems with time-delay from the sensor to the controller and/or from the controller to the actuator. It is widely known that time-delay arises pervasively in dynamic systems and very often is the cause for instability and poor performance of control systems (Gu, Kharitonov, & Jie, 2003). In the context of linear periodic systems with time-delay, Letyagina and Zhabko (2009) has studied the stability of continuous-time

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systems, whereas some results on asymptotic stability of discrete-time systems with a time-delayed state have been proposed in de Souza and Coutinho (2010), including robust stability criteria in the context of convex-bounded parameter uncertainty. However, it is hard to apply the later results for designing stabilizing controllers.

In this paper, we firstly consider the problem of robust stability analysis for linear discrete-time periodic systems subject to time-delay in the state variables and polytopic-type parameter uncertainty in the matrices of the state-space model. Both delay-independent and delay-dependent criteria of robust stability which are tailored for state feedback control synthesis are developed, where the former deals with a constant time-delay and the latter applies to the case of a varying time-delay lying in a given interval. The proposed stability conditions are based on affinely uncertainty-dependent Lyapunov–Krasovskii functionals and are cast via linear matrix inequalities (LMIs). The robust stability results are then adapted to solve problems of robust stabilization and robust H_∞ control via static periodic state feedback.

The paper is organized as follows. Section 2 introduces the class of systems studied in the paper and the formulation of the problems to be tackled. Section 3 develops methods of robust stability analysis. In Section 4, we derive techniques for designing robustly stabilizing static periodic state feedback controllers, whereas Section 5 proposes methods of robust H_∞ control. Numerical examples are presented throughout the paper to demonstrate the effectiveness of the proposed robust stability analysis and robust control methods. Finally, concluding remarks are drawn in Section 6.

Notation. $\mathbb{Z}$ is the set of integers, $\mathbb{Z}^+$ is the set of nonnegative integers, $\mathbb{R}^n$ is the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of $m \times n$ real matrices, I_n is the $n \times n$ identity matrix, 0_n is the $n \times n$ matrix of zeros, S^T denotes the transpose of S, and $\mathrm{diag}\{\cdots\}$ is a block-diagonal matrix. For symmetric block matrices, $\star$ stands for the transpose of the blocks outside of the main diagonal block. A matrix $S(k)$ is denoted N-periodic, where $0 < N \in \mathbb{Z}^+$, if $S(k+N) = S(k)$, for all $k \in \mathbb{Z}$. For a real N-periodic matrix, $S(k)$ nonsingular and $S(k) > 0$ ($S(k) \geq 0$) mean that $S(k)$ is respectively nonsingular and symmetric positive definite (positive semi-definite) for $k = 1, \ldots, N$. ℓ_2 denotes the space of squared summable vector sequences over $\mathbb{Z}^+$ with norm $\|x\|_2 := (\sum_{k=0}^{\infty} \|x(k)\|^2)^{\frac{1}{2}}$, where $\|\cdot\|$ is the Euclidean vector norm.

2. System model and preliminaries

Consider the following uncertain linear discrete-time periodic system:

$$\begin{cases} x(k+1) = A(k)x(k) + A_d(k)x(k-d(k)) + B(k)u(k) \\ x(k) = \phi(k), \quad k = -d_2, -d_2+1, \ldots, 0 \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^{n_u}$ is the control input, $d(k)$ is a time-delay, which is a positive integer to be specified in the sequel, $\phi(k)$ is the system initial sequence, d_2 is a given positive integer that represents the maximum allowed time-delay, and $A(k), A_d(k)$ and $B(k)$ are uncertain N-periodic real matrices with appropriate dimensions that are assumed to be confined to the following polytope:

$$\Omega(k) = \left\{ \Pi(k) : \Pi(k) = \sum_{i=1}^{\upsilon} \lambda_i \Pi_i(k),\ \lambda_i \geq 0,\ \sum_{i=1}^{\upsilon} \lambda_i = 1 \right\} \tag{2}$$

where

$$\Pi(k) = \begin{bmatrix} A(k) & A_d(k) & B(k) \end{bmatrix}, \tag{3}$$

$$\Pi_i(k) = \begin{bmatrix} A_i(k) & A_{di}(k) & B_i(k) \end{bmatrix}, \tag{4}$$

with $A_i(k), A_{di}(k)$ and $B_i(k)$, $i = 1, \ldots, \upsilon$, being given N-periodic real matrices. For notation simplicity, we denote

$$\Omega := \left\{ \Omega(k),\quad k = 1, \ldots, N \right\} \tag{5}$$

as the uncertainty polytope. Note that $\upsilon = 1$ corresponds to the case where system (1) is uncertainty free.

This paper aims at designing a stabilizing static state feedback for the system (1) as below

$$u(k) = K(k)x(k) \tag{6}$$

where $K(k)$ is an N-periodic matrix to be found such that the closed-loop system is asymptotically stable for all system matrices belonging to the uncertainty polytope Ω. In this case, the closed-loop system is said to be *Ω-robustly stable*. Both the settings of delay-independent and delay-dependent Lyapunov–Krasovskii stability conditions will be treated. For each of these cases, we shall consider different assumptions for the time-delay as follows:

Case 1 (*delay-independent stability*). The time-delay is assumed to be constant, namely $d(k) = d_2 > 0$, $\forall\, k \in \mathbb{Z}^+$.

Case 2 (*delay-dependent stability*). The time-delay $d(k)$ is time-varying and satisfies

$$d_1 \leq d(k) \leq d_2, \quad \forall\, k \in \mathbb{Z}^+ \tag{7}$$

where $d_2 > d_1 > 0$ are given integers.

The motivation for considering a time-varying delay is that this phenomenon often appears in a number of applications as, for instance, in multirate control of networked control systems where the time-delay is due to time-varying communication delays.

In this paper, we shall develop LMI based conditions for robust stabilization and robust H_∞ control for the Cases 1 and 2 as above. Firstly, we will derive LMI conditions for assessing the robust stability of the unforced system of (1), which are later applied for designing robust stabilizing and robust H_∞ periodic state feedback controllers.

3. Robust stability analysis

In de Souza and Coutinho (2010), the authors have derived delay-independent and delay-dependent LMI conditions to ascertain the robust stability of discrete-time periodic systems with time-delay. However, it turns out that it is hard to apply these conditions to design stabilizing state feedback. In the following, we introduce two new robust stability results for the system (1)–(4) for the Cases 1 and 2, which are tailored for state feedback robust control synthesis.

3.1. Delay-independent robust stability

Theorem 1. *Consider the uncertain system (1)–(4) with $u(k) \equiv 0$ and a given constant time-delay $d(k) = d_2$. Suppose there exist N-periodic matrices $G(k)$, $R_i(k) > 0$ and $W_i(k) > 0$, $i = 1, \ldots, \upsilon$, satisfying the following LMIs:*

$$\begin{bmatrix} \mathcal{U}_i(k) & \star & \star \\ 0 & -R_i(k-d_2) & \star \\ A_i(k)G(k) & A_{di}(k)G(k-d_2) & -W_i(k+1) \end{bmatrix} < 0,$$
$$k = 1, \ldots, N,\ i = 1, \ldots, \upsilon, \tag{8}$$

where

$$\mathcal{U}_i(k) = -G(k) - G^T(k) + W_i(k) + R_i(k).$$

Then, system (1) with $u(k) \equiv 0$ is Ω-robustly stable for any finite time-delay d_2.

Proof. See Appendix A.