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Nonlinear squeezing time–frequency transform for weak signal detection

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ABSTRACT

Conventional time–frequency analysis methods can characterize the time–frequency pattern of multi-component nonstationary signals. However, it is difficult to detect weak components hidden in complex signals because the time–frequency representation is influenced by the signal amplitude. In this paper, a novel algorithm called nonlinear squeezing time–frequency transform (NSTFT) is proposed to characterize the time–frequency pattern of multi-component nonstationary signals. Most importantly, theoretical analysis shows that the NSTFT method is independent of the signal amplitude and is only relevant to the signal phase, thus it can be used for weak signal detection. Moreover, an improved ridge detection algorithm is proposed in this paper for instantaneous frequency estimation. The experiments on simulated and real-world signals show that the NSTFT method can effectively detect weak components in complex signals, and the comparison study with some other time–frequency analysis methods also shows the advantages of the NSTFT method in weak signal detection.

1. Introduction

Time–frequency analysis (TFA) provides a powerful tool to characterize the time–frequency pattern of multi-component nonstationary signals. Time–frequency (TF) representations obtained by TFA methods can give insight into the complex structure of signals consisting of several components. TFA methods have been studied in a wide range of fields, including seismic [1,2], radar and sonar [3,4], communications [5,6], biomedicine [7,8], power system [9,10], mechanical engineering [11–15], etc.

The effectiveness of TFA methods depends on their energy concentration of TF representations, and thus affects the accuracy of instantaneous frequency (IF) estimation [16]. Some methods have been proposed to improve the energy concentration or the accuracy of the IF estimation. One way to improve the energy concentration is to obtain a good match between signals and atoms through devising TF atoms or demodulating signal, such as parametric TFA [17,18], adaptive short-time Fourier transform (STFT) [19,20], local polynomial Fourier transform [21,22], multi-view TF distribution [23], and matching demodulation transform [24,25]. On the other hand, post-processing TFA methods are investigated to improve the energy concentration, and the reassignment method is a typical example [26–28]. The reassignment of a TF representation transfers the value at any point to the gravity center of the signal's energy distribution [29]. The synchrosqueezing transform (SST) proposed by Daubechies et al. [30] uses the ratio of

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the time-shift derivative of signal's TF representation to the TF representation itself as the candidate IF to extract the IF information, and accordingly reassigns the signal energy distribution. Compared with other reassignment methods, the SST causes less deformation for IF profile and provides an exact reconstruction formula for constituent components [31–34].

All these methods can effectively improve the energy concentration of the TF representation. However, they may have some shortcomings for the detection of weak segments in mono-component nonstationary signals or weak components in multi-component nonstationary signals because the TF representation is influenced by the signal amplitude. Reviewing the intermediate procedure in the SST, it can be found that the ratio between the two STFT representations can reveal the IF information. Most importantly, this ratio is only relevant to the signal phase and is independent with the signal amplitude, and the ratio is actually the log-derivative of a STFT and is used in reassignment-like methods [35,36].

Inspired by the calculation procedure of candidate IF in the SST, nonlinear squeezing time–frequency transform (NSTFT) is proposed to characterize the TF pattern of multi-component nonstationary signals, especially for weak signal detection, including weak segments in mono-component signals and weak components in multi-component signals. Compared with the SST using reassignment strategy, NSTFT combines two TF representations to highlight the coefficient at the IF and to squeeze the coefficient around the IF. Theoretical analysis shows that, because of the interaction between the two TF representations, the NSTFT method is only relevant to the signal phase and is independent of the signal amplitude, thus the influence caused by the signal amplitude can be reduced, and the NSTFT method can be used for weak signal detection. The application in some real-world signals, including a bat echolocation signal and a vibration signal collected from a heavy oil catalytic cracking machine set, shows that the NSTFT can be effective in weak signal detection.

The remainder of this paper is organized as follows. In Section 2, the basic theoretical background concerning STFT and SST is reviewed. Section 3 describes the proposed NSTFT method for weak signal detection, and the theoretical analysis is also provided. Moreover an improved ridge extraction method is also introduced for IF estimation. Section 4 uses some simulated and real-world signals to verify the performance of the proposed NSTFT method in weak signal detection. Finally, several conclusions are drawn in Section 5.

2. Theoretical background

2.1. FM signal and instantaneous frequency

Multi-component frequency-modulation (FM) signals can be modeled with sums of sinusoidal functions, i.e. the signal $x(t)$ can be represented by the following model:

$$x(t)=\sum_{k=1}^{K}x_k(t), \tag{1}$$

with

$$x_k(t)=A_k(t)\cos\left(\phi_k(t)\right) \text{ and } A_k(t)>0, \quad \phi_k'(t)>0, \tag{2}$$

where the amplitude $A_k(t)$ and the phase $\phi_k(t)$ are defined in terms of the analytic signal $z_k(t)$. In this paper, the analytic signal $z_k(t)$ is given by Hilbert transform as follows:

$$z_k(t)=x_k(t)+i\mathscr{H}[x_k(t)], \tag{3}$$

where the Hilbert transform of $x_k(t)$ is defined as

$$\mathscr{H}[x_k(t)]=\pi^{-1}\text{P.V.}\int_{-\infty}^{+\infty}\frac{x_k(\tau)}{t-\tau}\,d\tau, \tag{4}$$

where P.V. means that the integral is taken in the sense of the Cauchy principal value.

If each phases $\phi_k(t)$ is a Blaschke product and the amplitude $A_k(t)$ and the phase $\phi_k(t)$ satisfy the Bedrsian identity, the construction of the analytic signal $z_k(t)$ permits $A_k(t)$ and $\phi_k(t)$ to be uniquely defined as

$$A_k(t)e^{i\phi_k(t)}=z_k(t), \tag{5}$$

and the original signal is recovered by $x_k(t)=\Re\{z_k(t)\}$. The instantaneous angular frequency of the component is the first derivative of the phase

$$\omega_k(t)=\phi_k'(t). \tag{6}$$

Typically, the changes of $A_k(t)$ and $\phi_k'(t)$ are much slower than the change of $\phi_k(t)$ itself, which means that locally the component $x_k(t)$ can be regarded as a harmonic signal with amplitude $A_k(t)$ and frequency $\phi_k'(t)$.

In this model, if $K=1$, the signal can be referred to as mono-component signal; if $K\geq 2$, the signal is referred to as multi-component signal. The model defined by (1)–(5) applies for multi-component signals and allows the modeling of$K$time-varying frequency laws.

2.2. STFT and synchrosqueezing transform

In this study, we use the modified STFT which is the regular STFT with a modulation factor instead of the regular STFT. The modified STFT of the signal $x\in\mathbf{L}^2(\mathbb{R})$ is defined as

$$S_x(u,\xi)=\int_{-\infty}^{+\infty}x(t)g_\sigma(t-u)e^{-i\xi(t-u)}\,dt, \tag{7}$$

where $g_\sigma(t)$ is a parametric form of the window function $g(t)$ as

$$g_\sigma(t)=\frac{1}{\sqrt{\sigma}}g\left(\frac{t}{\sigma}\right). \tag{8}$$

Synchrosqueezing transform (SST) was originally introduced in the context of audio signal analysis. Like empirical mode decomposition (EMD) [37,38], synchrosqueezing can extract and clearly depict components with time-varying spectrum. STFT-based SST is performed in three steps.

First, the STFT $S_x(u,\xi)$ of the signal is calculated in (7). We use a purely harmonic signal $x(t)=A\,e^{i2\pi f_0 t}$ for motivation. For the purely harmonic signal $x(t)$, according to