Automatica 50 (2014) 513–518

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

On-line fault detection and isolation for linear discrete-time uncertain systems☆

Ze Zhang, Imad M. Jaimoukha [1]

Control and Power Group, Department of Electrical and Electronic Engineering, Imperial College, London SW7 2AZ, UK

ARTICLE INFO

Article history:
Received 23 February 2012
Received in revised form
29 May 2013
Accepted 8 October 2013
Available online 2 December 2013

Keywords:
Fault detection and isolation
Structured uncertainty
Linear discrete-time system
Linear matrix inequalities

ABSTRACT

This work proposes a robust fault detection and isolation (FDI) scheme for linear discrete-time systems subject to faults, bounded additive disturbances and norm-bounded structured uncertainties. FDI is achieved by computing, on-line, upper and lower bounds on the fault signal such that a fault is regarded as having occurred when its upper bound is smaller than zero or lower bound is larger than zero. Linear Matrix Inequality (LMI) optimization techniques are used to obtain the bounds. Furthermore, a subsequent-state-estimation technique, together with an estimation horizon update procedure, is proposed, which allows the on-line FDI process to be repeated in a moving horizon procedure. The approach is also extended to solve the fault detection (FD) problem of obtaining lower bounds on the total fault signal energy within the estimation horizon. The scheme gives the best estimates of the fault signals given the information available and is sufficiently flexible to incorporate other information that may be available, such as bounds on the disturbance energy. Thus our scheme is immune to false alarms if the system and disturbance are within the uncertainty description. Moreover, we propose a new robustness result to obtain the bounds, which is an extension of current techniques for handling model uncertainties. Finally, the approach is verified using two numerical examples.

1. Introduction

With the increasing requirements on control system reliability and security, much attention has been devoted to the design of FDI systems (Jaimoukha, Li, & Papakos, 2006; Li & Jaimoukha, 2009; Li, Mazars, Zhang, & Jaimoukha, 2012; Ye, Wang, & Ding, 2004; Zhang & Ding, 2008; Zhang & Jaimoukha, 2009; Zhong, Ding, Lam, & Wang, 2003; Zhong, Ding, Tang, Zhang, & Jeinsch, 2001). There are two main approaches for FDI: observer-based and parity space approaches.

Observer-based approaches exploit analytic redundancy and use a mathematic model of the system to design an observer generating residual signals that provide fault signatures (Mazars, Jaimoukha, & Li, 2008; Zhong et al., 2003, 2001). The observer effectively cancels the (nominal) process dynamics and generates a residual signal which is sensitive only to disturbances, plant/model mismatch (often recast as disturbances) and faults. The design objective is then transformed to a sensitivity optimization problem, which seeks to increase the sensitivity of the residual to faults and simultaneously reduce the sensitivity to disturbances and plant/model mismatch. However, a drawback of these approaches is that their effectiveness depends on the accuracy of the system model, which cannot usually be guaranteed in practice.

In parity space approaches, a residual generator is designed by computing, off-line, a parity vector or matrix which completely decouples the system state and improves the sensitivity of the residual to faults and robustness to disturbances (Ye et al., 2004; Zhang & Ding, 2007; Zhong, Ding, & Shi, 2009). Input and output measurements are then collected over an estimation horizon to generate the residual. Since parity-space-based FD design only involves vector or matrix valued mathematical operations, it has attracted research interest and wide attention from industry for applications (Zhang & Ding, 2008).

In this paper, an on-line approach is proposed to solve robust FDI problems for linear discrete-time systems subject to faults, bounded additive disturbances and norm-bounded structured uncertainties. This scheme uses a dynamic system model as well as input/output measurements over an estimation horizon to compute upper and lower bounds on the fault signals at each sampling instant. Then a fault is regarded as having occurred when its upper bound is smaller than zero or lower bound is larger than zero. LMI optimization techniques are employed to obtain the bounds. Furthermore, a subsequent-state-estimation technique together with an estimation horizon update procedure is proposed,

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Yasumasa Fujisaki under the direction of Editor Roberto Tempo.

E-mail addresses: ze.zhang06@imperial.ac.uk (Z. Zhang), i.jaimouka@imperial.ac.uk (I.M. Jaimoukha).

[1] Tel.: +44 2075946279; fax: +44 2075946282.

0005-1098/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.automatica.2013.11.003

which allows the on-line FDI process to be repeated in a moving horizon procedure. Also, the proposed approach reduces to the solution of a set of linear programs when there are no uncertainties in the model description and it is shown that, compared with solving FDI problems, this approach gives better reliability and fault detectability for the FD problem. Preliminary versions of our approach, which only considered restricted classes of uncertainty, have appeared in Zhang and Jaimoukha (2009) and Zhang and Jaimoukha (2010).

In common with the parity space approach, we use a dynamic system model as well as input/output measurements over an estimation horizon. The main difference is that, instead of using a pre-computed parity vector to define a residual signal, we compute bounds on the faults by solving on-line optimization problems subject to the constraints of the bounds on the initial state, disturbances and norm-bounded structured uncertainties. One feature of our approach is that available bounds on the initial state can be used and the initial state does not need to be decoupled, which is an advantage especially for uncertain systems.

This work is organized as follows. After defining the notation, Section 2 defines the problem setting. Section 3, of independent interests, develops a new and general robustness tool for handling the types of model uncertainties considered in this paper. Section 4 gives the problem formulation and derives bounds on the fault signals in the form of solutions to LMI problems. In Section 5, an overall FDI algorithm is presented and its properties are investigated. Two examples from the literature are given in Section 6 to demonstrate the effectiveness of the proposed scheme. Finally, Section 7 summarizes this work.

The notation is fairly standard. $\mathcal{R}^{n\times m}$ denotes the set of real $n \times m$ matrices. For $A \in \mathcal{R}^{n\times m}$, A^T denotes the transpose. The ith eigenvalue of $A \in \mathcal{R}^{n\times n}$ is denoted by $\lambda_i(A)$. For $A = A^T, A \succeq 0$ ($A \preceq 0$) denotes that A is positive (negative) semidefinite, that is, $\lambda_i(A) \geq 0$ (≤ 0) $\forall\, i$. For $x, y \in \mathcal{R}^n, x < y$ and $x \leq y$ denote element-wise inequalities. If $\mathcal{S} \subseteq \mathcal{R}^{n\times m}$ is a subspace, $\mathcal{B}\mathcal{S} = \{S \in \mathcal{S} : \|S\| \leq 1\}$ denotes the unit ball of $\mathcal{S}$ where $\|S\| = \max_i \lambda_i(SS^T)$ denotes the matrix norm. I_n denotes the $n \times n$ identity matrix and $0_{n\times m}$ denotes the $n \times m$ null matrix with the subscripts dropped if they can be inferred from context. $\mathcal{S}^n$, $\mathcal{S}^n_+$ and $\mathcal{D}^n_+$ denote the sets of all real $n\times n$ symmetric, positive semidefinite and positive semidefinite diagonal matrices, respectively.

2. FDI problem setting

Consider a linear time-invariant (LTI) discrete-time system subject to disturbances, process, actuator and sensor faults and uncertainties of the form

$$\begin{aligned} x_{k+1} &= Ax_k + B_d d_k + B_f f_k + B_u u_k + B_p p_k \\ y_k &= C_y x_k + D_{yd} d_k + D_{yf} f_k + D_{yu} u_k + D_{yp} p_k \\ q_k &= C_q x_k + D_{qd} d_k + D_{qf} f_k + D_{qu} u_k \\ p_k &= \Delta_0 q_k \end{aligned} \tag{1}$$

for $k \in \mathcal{N}$, where $\mathcal{N} := \{0, \ldots, N-1\}$ is the estimation horizon, $x_k \in \mathcal{R}^n, u_k \in \mathcal{R}^{n_u}$ and $y_k \in \mathcal{R}^{n_y}$ are the state, input, output and uncertainty vectors, respectively, $p_k, q_k \in \mathcal{R}^{n_\Delta}$ are the input and output uncertainty vectors, respectively and $d_k \in \mathcal{R}^{n_d}$ and $f_k \in \mathcal{R}^{n_f}$ are the disturbance and fault vectors, respectively. Also given are the system, fault, disturbance and uncertainty distribution matrices of appropriate dimensions (Frank & Ding, 1997). The uncertainty matrix Δ_0 is assumed to be a norm-bounded structured matrix, $\Delta_0 \in \mathcal{B}\mathbf{\Delta_0}$ where $\mathbf{\Delta}_0 \subseteq \mathcal{R}^{n_\Delta\times n_\Delta}$ is a structured subspace, typically block diagonal with full and repeated blocks; see Zhou, Doyle, and Glover (1996) for a description.

We also assume that the initial state and disturbances are unknown, but that upper and lower bounds $\bar{x}_0, \underline{x}_0, \bar{d}_k$ and $\underline{d}_k$ are available so that $\underline{x}_0 \leq x_0 \leq \bar{x}_0$ and $\underline{d}_k \leq d_k \leq \bar{d}_k$ for all $k \in \mathcal{N}$. Also assume that measurements of the input and output signals u_k and y_k are available for all $k \in \mathcal{N}$. Define $x = [x_1^T \ \cdots \ x_N^T]^T$ and $\xi = [\xi_0^T \ \cdots \ \xi_{N-1}^T]^T \in \mathcal{R}^{N_\xi}$ where $N_\xi = N \cdot n_\xi$ with ξ standing for $d, \bar{d}, \underline{d}, f, p, q, u$ and y, take $n_p = n_q = n_\Delta$ and $n_{\bar{d}} = n_{\underline{d}} = n_d$. Using an iteration,

$$\begin{aligned} x &= \mathcal{A}x_0 + \mathcal{B}_d d + \mathcal{B}_f f + \mathcal{B}_u u + \mathcal{B}_p p \\ y &= \mathcal{C}_y x_0 + \mathcal{D}_{yd} d + \mathcal{D}_{yf} f + \mathcal{D}_{yu} u + \mathcal{D}_{yp} p \\ q &= \mathcal{C}_q x_0 + \mathcal{D}_{qd} d + \mathcal{D}_{qf} f + \mathcal{D}_{qu} u + \mathcal{D}_{qp} p, \\ p &= \Delta q, \quad \Delta \in \mathcal{B}\mathbf{\Delta}, \quad \mathbf{\Delta} = I_N \otimes \mathbf{\Delta}_0 \subset \mathcal{R}^{N_\Delta\times N_\Delta} \end{aligned} \tag{2}$$

where $\otimes$ denotes the Kronecker product, $N_\Delta = Nn_\Delta$ and

$$\begin{aligned} \mathcal{A} &= \begin{bmatrix} A \\ \vdots \\ A^{N-1} \end{bmatrix}, \quad \mathcal{B}_\zeta = \begin{bmatrix} B_\zeta & \cdots & 0 \\ \vdots & \ddots & \vdots \\ A^{N-1}B_\zeta & \cdots & B_\zeta \end{bmatrix} \\ \mathcal{C}_\beta &= \begin{bmatrix} C_\beta \\ \vdots \\ C_\beta A^{N-1} \end{bmatrix}, \quad \mathcal{D}_{\beta\zeta} = \begin{bmatrix} D_{\beta\zeta} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ C_\beta A^{N-2}B_\zeta & \cdots & D_{\beta\zeta} \end{bmatrix} \end{aligned} \tag{3}$$

where β stands for q and y and ζ for d, f, u and p, and note that $D_{qp} = 0$. To simplify the presentation, define

$$\begin{aligned} &w = [x_0^T \ d^T]^T, \quad \underline{w} = [\underline{x}_0^T \ \underline{d}^T], \quad \bar{w} = [\bar{x}_0^T \ \bar{d}^T], \\ &\mathcal{B}_w = [\mathcal{A} \ \mathcal{B}_d], \quad \mathcal{D}_{yw} = [\mathcal{C}_y \ \mathcal{D}_{yd}], \quad \mathcal{D}_{qw} = [\mathcal{C}_q \ \mathcal{D}_{qd}], \end{aligned}$$

and $N_w = n + N_d$ and note that

$$\underline{w} \leq w \leq \bar{w}. \tag{4}$$

Eliminating p from (2) using $p = \Delta q$ gives

$$x = \mathcal{B}_w^\Delta w + \mathcal{B}_f^\Delta f + \mathcal{B}_u^\Delta u, \qquad y = \mathcal{D}_{yw}^\Delta w + \mathcal{D}_{yf}^\Delta f + \mathcal{D}_{yu}^\Delta u \tag{5}$$

where $\mathcal{B}_\zeta^\Delta = \mathcal{B}_\zeta + \mathcal{B}_p\hat{\Delta}\mathcal{D}_{q\zeta}$ and $\mathcal{D}_{y\zeta}^\Delta = \mathcal{D}_{y\zeta} + \mathcal{D}_{yp}\hat{\Delta}\mathcal{D}_{q\zeta}$ and where $\hat{\Delta} := \Delta(I - \mathcal{D}_{qp}\Delta)^{-1}$ and ζ stands for w, f and u.

3. Robustness results for structured uncertainty

Since the FDI system we consider includes norm-bounded structured uncertainties, in this section we derive a new robustness result for matrix inequality constraints that involve a quadratic term in the uncertainty. This will be used later when we evaluate the bounds on the fault signals. The following well-known lemma is needed for our robustness results.

Lemma 1 (*Boyd, El Ghaoui, Feron, & Balakrishnan, 1994*). *Let A_0, $A_1 \in \mathcal{S}^n$ and assume that $z^TA_1z > 0$ for some z. Then $x^TA_0x \geq 0$ for all x satisfying $x^TA_1x \geq 0$ if and only if there exists $0 \leq \tau \in \mathcal{R}$ such that $A_0 - \tau A_1 \succeq 0$.*

The next robustness theorem for unstructured uncertainty is a corollary of the above result.

Theorem 2. *Let $T_1 \in \mathcal{S}^n, T_2 \in \mathcal{R}^{n\times m}, T_3 \in \mathcal{R}^{m\times n}, T_4 \in \mathcal{R}^{m\times m}$, $T_5 \in \mathcal{S}^m$ and let $\mathbf{\Delta} = \mathcal{R}^{m\times m}$. Then $\det(I - T_4\Delta) \neq 0$ and $T(\Delta) := T_1 + T_2\hat{\Delta}T_3 + T_3^T\hat{\Delta}^TT_2^T + T_2\hat{\Delta}T_5\hat{\Delta}^TT_2^T \succeq 0$ for all $\Delta \in \mathcal{B}\mathbf{\Delta}$, where $\hat{\Delta} := \Delta(I - T_4\Delta)^{-1}$, if and only if $\|T_4\| < 1$ and there exists $0 \leq \tau \in \mathcal{R}$ such that*

$$\begin{bmatrix} T_1 - \tau T_2T_2^T & T_3^T - \tau T_2T_4^T \\ \star & T_5 + \tau(I - T_4T_4^T) \end{bmatrix} \succeq 0 \tag{6}$$

where $\star$ denotes terms readily deduced from symmetry.