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Brief paper

Adaptive observers design for a class of linear descriptor systems[☆]

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ABSTRACT

In this paper, we propose a new approach to an adaptive observer design for a class of multi-input–multi-output linear descriptor systems. Global exponential convergence for joint state-parameter estimation is established for noise-free systems. For noise corrupted descriptor systems, it is proved that the state and parameter estimation errors are bounded and converge in the mean to zero, when the noises are bounded and have zero means. Potential applications of the presented adaptive observer are online system identification, fault detection and adaptive control of descriptor systems. Numerical example is presented to illustrate the performance of the proposed adaptive observer.

1. Introduction and problem formulation

The problem of the observer design has been a very active field of research during the last decades. This is due to the fact that the state estimation is generally required for the control and diagnosis when all the states of the system are not available. When all or part of the parameters are unknown, they can be estimated by using a joint estimation of state and parameter observers known as adaptive observers. For standard systems there exist many results on the adaptive observer design (see Bastin & Gevers, 1988, Besançon, 2000, Besançon, De Leon-Morales, & Huerta-Guevara, 2006, Marino & Tomei, 1995 and references therein). On the other hand descriptor systems also known as generalized, singular or differential algebraic systems describe a large class of systems for which the standard state space representation is not applicable. They are encountered in chemical, mineral, electrical and mechanical systems (Darouach & Zasadzinski, 1991; Lewis, 1986; Liu, Zhang, Yang, & Yang, 2008; Luenberger, 1979). Recently, descriptor systems have been used to model biological complex systems. Observer design for linear descriptor systems has been greatly treated and there exist many results on the full and reduced order observers design. However, contrary to standard systems where the adaptive observer design problem has been widely studied, there exist only few results on an adaptive observer design for descriptor systems (Uetake, 1994). The purpose of the present paper is to propose a new method for the adaptive observer design for a class of linear descriptor systems. The approach is based on the one developed in Darouach and Boutayeb (1995), where only the state is estimated; here also we consider the estimation of unknown constant parameters. The conditions for the existence of these observers are also derived and as in Zhang (2002), the stability and the convergence of the proposed algorithm is presented.

The paper is organized as follows. The considered problem is presented and formulated in Section 1. In Section 2, the proposed adaptive observer for a class of linear descriptor systems is presented. The stability and the convergence of the algorithm for the noise corrupted systems is given in Section 3. In Section 4, an illustrative numerical example is presented. Finally, some concluding remarks are drawn in Section 5.

Consider the following linear time invariant descriptor system:

$$E^*\dot{x}(t) = A^*x(t) + B^*u(t) + \psi^*(t)\theta + w^*(t)$$
$$y^*(t) = C^*x(t) + v^*(t) \tag{1}$$

where $x \in R^n$, $u \in R^k$, and $y^* \in R^p$ are the state vector, the input vector, and the output vector, respectively. The vector $\theta \in R^l$ is the unknown parameter vector assumed constant. $E^* \in R^{m\times n}$, $A^* \in R^{m\times n}$, $B^* \in R^{m\times k}$, and $C^* \in R^{p\times n}$ are known constant matrices. $\psi^*(t) \in R^{m\times l}$ is a matrix of known signals, it is assumed to

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be piecewise differentiable and $\psi^*(t)$ and $\dot{\psi}^*(t)$ are uniformly bounded in time. Signals $w^*(t) \in R^n$ and $v^*(t) \in R^p$ represent the state and observation noises, assumed to be bounded with zero mean, respectively. We assume that: rank $E^* = r < n$, and without loss of generality: rank $C^* = p$.

The considered problem in this paper is the joint estimation of $x(t)$ and θ from known signals $u^*(t)$ and $y^*(t)$, respectively.

In the sequel we assume that:

$$\text{rank}\begin{bmatrix} E^* & A^* \\ 0 & E^* \\ 0 & C^* \end{bmatrix} = n + \text{rank}\, E^*. \tag{2}$$

Then, since rank $E^* = r$, there exists a nonsingular matrix P such that:

$$PE^* = \begin{bmatrix} E \\ 0 \end{bmatrix}, \qquad PA^* = \begin{bmatrix} A \\ A_1 \end{bmatrix} \tag{3}$$

$$PB^* = \begin{bmatrix} B \\ B_1 \end{bmatrix} \quad \text{and} \quad P\psi^*(t) = \begin{bmatrix} \psi(t) \\ \psi_1(t) \end{bmatrix} \tag{4}$$

with $E \in R^{r\times n}$ and rank $E = r$.

And

$$Pw^*(t) = \begin{bmatrix} w(t) \\ w_1(t) \end{bmatrix}. \tag{5}$$

Following Dai (1989), system (1) is a restricted system equivalent to:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) + \psi(t)\theta + w(t) \\ y(t) &= Cx(t) + \psi_2(t)\theta + v(t) \end{aligned} \tag{6}$$

where

$$y(t) = \begin{bmatrix} -B_1 u(t) \\ y^*(t) \end{bmatrix} \in R^q$$

$$C = \begin{bmatrix} A_1 \\ C^* \end{bmatrix} \in R^{q\times n}$$

and

$$\psi_2(t) = \begin{bmatrix} \psi_1(t) \\ 0 \end{bmatrix} \in R^{q\times l},$$

and

$$v(t) = \begin{bmatrix} w_1(t) \\ v^*(t) \end{bmatrix} \in R^q,$$

with $q = m + p - r$.

Then using the above transformations, we can prove that condition (2) is equivalent to:

$$\text{rank}\begin{bmatrix} E \\ C \end{bmatrix} = n \tag{7}$$

since E is a full row rank matrix.

It follows that there exists a nonsingular matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that:

$$\begin{aligned} aE + bC &= I_n \\ cE + dC &= 0. \end{aligned} \tag{8}$$

Our aim is to design an adaptive observer of a fixed order n satisfying:

$$\begin{aligned} \dot{z} &= Nz + L_1 y_2 + L_2 y_2 + Gu + (a + Kc)\psi\hat{\theta} + \Upsilon\dot{\hat{\theta}} \\ \hat{x} &= z + by_2 + Kdy_2 \\ \dot{\hat{\theta}} &= \Gamma\gamma^T C^{*T}\Sigma(y^* - \hat{y}^*) \end{aligned} \tag{9}$$

where $\hat{x}(t), \hat{\theta}(t)$ are the estimates of $x(t)$ and $\theta(t)$ respectively, with:

$$\begin{aligned} y_2(t) &= y(t) - \psi_2(t)\hat{\theta}(t) \\ \hat{y}^* &= C^*\hat{x}(t) \end{aligned} \tag{10}$$

and $N, L_1, L_2, G, K, \Upsilon(t)$ and $\gamma(t)$ are unknown matrices of appropriate dimensions, which must be determined such that $\hat{x}(t)$ and $\hat{\theta}(t)$ will converge to $x(t)$ and θ, respectively.

2. Adaptive observers design

Consider the case, where the system (1) is noise free, i.e., $w^*(t) = 0$ and $v^*(t) = 0$. It follows that: $w(t) = 0$ and $v(t) = 0$.

Now, we can define the two estimate errors $e_1(t)$ and $e_2(t)$ such that:

$$\begin{aligned} e_1(t) &= x(t) - \hat{x}(t) \\ e_2(t) &= \theta - \hat{\theta}(t). \end{aligned} \tag{11}$$

From the above equations, $y_2(t)$ can be written as:

$$y_2(t) = Cx(t) + \psi_2(t)e_2(t). \tag{12}$$

From (6), (8) and (9), it is easy to show that the state estimate error is given by:

$$\begin{aligned} e_1 &= x - \hat{x} \\ &= (a + Kc)Ex - z - (b + Kd)\psi_2 e_2 \end{aligned} \tag{13}$$

its derivative is:

$$\begin{aligned} \dot{e}_1 &= Ne_1 + [(a + Kc)A - N(a + Kc)E - L_1 C - L_2 C]x \\ &\quad + [(a + Kc)B - G]u - [(b + Kd)\psi_2 - \Upsilon]\dot{e}_2 \\ &\quad + [(a + Kc)\psi - L_1\psi_2 - L_2\psi_2 \\ &\quad + N(b + Kd)\psi_2 - (b + Kd)\dot{\psi}_2]e_2. \end{aligned} \tag{14}$$

Now, we can give the following assumptions which are necessary for the rest of the paper.

Assumption 1. Assume that Darouach and Boutayeb (1995):

$$\text{rank}\begin{bmatrix} E^* & A^* \\ 0 & E^* \\ 0 & C^* \end{bmatrix} = n + \text{rank}\, E^* \tag{15}$$

and

$$\text{rank}\begin{bmatrix} sE^* - A^* \\ C^* \end{bmatrix} = n, \quad \forall s \in C,\ \text{Re}(s) \geq 0. \tag{16}$$

Assumption 2. Let $\gamma(t) \in R^{n\times l}$ be a matrix of signals generated by the ODE stable system:

$$\dot{\gamma}(t) = N\gamma(t) + \chi(t) \tag{17}$$

where:

$$\chi(t) = \begin{bmatrix} -(a + Kc) & L_2 \end{bmatrix}\begin{bmatrix} \psi(t) \\ \psi_2(t) \end{bmatrix} + (b + Kd)\dot{\psi}_2(t). \tag{18}$$

Assume that $\chi(t)$ is persistently exciting so that there exist two positive constants δ, T and some bounded symmetric positive definite matrix $\Sigma(t) \in R^{q\times q}$ such that for all t the following inequality holds:

$$\int_t^{t+T} \gamma^T(\tau)C^T\Sigma C\gamma(\tau)d\tau \geq \delta I. \tag{19}$$

Remark 1. Condition (16) is equivalent to the pair $(aA, \begin{bmatrix} cA \\ C \end{bmatrix})$ is detectable. In this case, two matrices α and β can be determined such that: $aA + \begin{bmatrix} \alpha & \beta \end{bmatrix}\begin{bmatrix} cA \\ C \end{bmatrix}$ is a stability matrix.