Automatica 50 (2014) 607–613

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Jet bundle formulation of infinite-dimensional port-Hamiltonian systems using differential operators☆

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ARTICLE INFO

Article history:
Received 10 December 2012
Received in revised form
5 August 2013
Accepted 4 November 2013
Available online 28 December 2013

Keywords:
Port-Hamiltonian systems
Differential geometry
Infinite-dimensional systems
Partial differential equations
System theory

ABSTRACT

We consider infinite-dimensional port-Hamiltonian systems described on jet bundles. Based on a power balance relation we introduce the port-Hamiltonian system representation using differential operators regarding the structural mapping, the dissipation mapping and the input mapping. In contrast to the well-known representation on the basis of the underlying Stokes–Dirac structure our approach is not necessarily based on using energy-variables which leads to a different port-Hamiltonian representation of the analyzed partial differential equations. The presented constructions will be specialized to mechanical systems to which class also the presented examples belong.

1. Introduction

Modeling of physical systems described by partial differential equations (pdes) in a port-Hamiltonian framework has been treated extensively over the last years. However, there are still many open issues and there exists no unique representation. An important contribution in this field was the introduction of Stokes–Dirac structures which allows us to analyze field theories in a port-Hamiltonian framework and to exploit this system representation for the controller design; see e.g. Macchelli and Melchiorri (2004b), Macchelli, van der Schaft, and Melchiorri (2004a,b), Maschke and van der Schaft (2005), and van der Schaft and Maschke (2002). Roughly speaking, the key property of the Stokes–Dirac structure is to represent the power balance relation of physical systems in a systematic way. This is achieved by combining the so-called flows and efforts in the domain and on the boundary of a system. Using this approach the proper choice of energy variables is crucial and the Hamiltonian is also a functional depending on energy variables. One key feature of the Stokes–Dirac approach is the fact that non-zero energy flow through the boundary can be considered, which is in contrast to approaches in mathematical physics where mainly the case of infinite domain or zero boundary conditions is under investigation.

A different kind of a port-Hamiltonian representation also allowing for non-zero energy flow through the boundary, not focusing on energy variables, but also based on a power conservation law, was proposed in Ennsbrunner and Schlacher (2005), Schlacher (2008) and Schöberl, Ennsbrunner, and Schlacher (2008) using the framework of jet bundles. This approach can be seen as an extension of Olver (1986) by incorporating dissipation and boundary ports and it has been exploited for control issues in Schöberl and Siuka (2013b), Siuka (2011) and Siuka, Schöberl, and Schlacher (2011). In this contribution we will present an enhancement of Ennsbrunner and Schlacher (2005), Schlacher (2008) and Schöberl et al. (2008) in such a way that besides the input map as in Schöberl et al. (2008) also the structural mapping and the dissipation mapping may involve differential operators. We will specialize our results for the class of mechanical port-Hamiltonian systems that allow for a variational principle. It should be noted that we apply the formal theory of pdes; this means that (geometric) properties of the equations are treated separately from the functional analytic properties of the solutions, where in this contribution we adopt the geometric point of view.

☆ The material in this paper was partially presented at the 4th IFAC Workshop on Lagrangian and Hamiltonian Methods for Non Linear Control (LHMNLC 2012), August 29–31, 2012, Bertinoro, Italy. This paper was recommended for publication in revised form by Associate Editor George Weiss under the direction of Editor Miroslav Krstic.

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0005-1098/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.automatica.2013.11.035

In the approach based on Stokes–Dirac structures it is quite natural that differential operators appear (especially the interconnection mapping), since energy variables are used in contrast to the approach presented in this paper. For instance in mechanics, the choice of energy variables suggests to use the strain ϵ as the dependent coordinate (state variable), whereas also the displacement (configuration) w is possible. This latter choice leads to derivative coordinates when the potential energy has to be stated, since the energy is a function of the strain and $\epsilon = \partial_X w$ is met in the case of a one-dimensional domain. For mechanical systems we believe that it is desirable to be able to describe the configuration explicitly and that it also should be possible to include forces that depend on the configuration (or that can be derived by a potential dependent on the configuration) which is difficult when energy variables are used. The choice of variables also has severe consequences in the application of the variational derivative applied to the Hamiltonian density, leading to different port-Hamiltonian representations, and has also an impact on the computation of structural invariants; see also Schöberl and Siuka (2013b). Also the generalization to mechanical systems with spatial domains greater than one is not so straightforward when the strains are used as state variables, since compatibility conditions arise; see Schöberl and Siuka (2013a) for a discussion.

This paper is organized as follows. In Section 2 some notation is presented and the geometric objects that play a fundamental role in the paper are introduced. Section 3 deals with the representation of port-Hamiltonian systems described by pdes, where the novelty in this paper is the fact that differential operators are considered, which is not the case in Ennsbrunner and Schlacher (2005) and Schlacher (2008). Two specific applications, the Timoshenko beam including damping effects and a simple model of magnetohydrodynamics are analyzed in Section 4 to demonstrate how the introduced differential operators have an impact on the power balance in practice. A preliminary version of the material has been presented in Schöberl and Siuka (2012).

2. Notation and preliminaries

In this paper we will apply differential geometric methods and we will use a notation that is similar to the one in Giachetta, Mangiarotti, and Sardanashvily (1997). To keep the formulas short and readable we will use tensor notation and especially Einstein's convention on sums. We use the standard symbol $\otimes$ for the tensor product, $\wedge$ denotes the exterior product (wedge product), d is the exterior derivative, and $\rfloor$ is the natural contraction between tensor fields. By ∂^B_α are meant the partial derivatives with respect to coordinates with the indices $\overset{\alpha}{B}$. Furthermore, $C^\infty(\cdot)$ denotes the set of the smooth functions on the corresponding manifold. Moreover, we will not indicate the range of the used indices when they are clear from the context. Additionally, pull-back bundles (see e.g. Giachetta et al., 1997) are not indicated when they follow from the context to avoid exaggerated notation.

In the following sections we will consider bundle structures in order to be able to separate dependent and independent coordinates. Let us consider the bundle $\mathcal{X} \to \mathcal{D}$ with coordinates (X^A, x^α) for $\mathcal{X}$ and (X^A) for $\mathcal{D}$ where x are the dependent and X the independent coordinates. The first jet manifold $\mathcal{J}^1(\mathcal{X})$ can be introduced possessing the coordinates $(X^A, x^\alpha, x^\alpha_A)$, where the capital Latin indices $A, B, \ldots$ are used for the base manifold $\mathcal{D}$ (independent coordinates) and x^α_A denote derivative coordinates of first order (derivatives of the dependent coordinates with respect to the independent ones), and $\partial_A = \partial/\partial X^A$, $\partial_\alpha = \partial/\partial x^\alpha$, $\partial^A_\alpha = \partial/\partial x^\alpha_A$ is met. Higher-order jet manifolds can be introduced accordingly by considering the space $\mathcal{J}^r(\mathcal{X})$ equipped with coordinates $(X^A, x^\alpha_{\bar{J}})$ where $\bar{J}$ is an unordered multi-index (modulo permutations) with $0 \le \#\bar{J} \le r$ where $\#\bar{J}$ characterizes the number of derivations; see Giachetta et al. (1997).

The jet structure induces the so-called total derivative $d_A = \partial_A + x^\alpha_{\bar{J}A}\partial^{\bar{J}}_\alpha$, $\#\bar{J} \ge 0$, e.g. in the case where it acts on elements living in $\mathcal{J}^1(\mathcal{X})$ we obtain $d_A = \partial_A + x^\alpha_A\partial_\alpha + x^\alpha_{AB}\partial^B_\alpha$. We will treat the so-called densities in the sequel (a quantity that can be integrated), where we pay special attendance to densities of the form $\mathfrak{F} = \mathcal{F}\Omega$ with $\mathcal{F} \in C^\infty(\mathcal{J}^1(\mathcal{X}))$ where Ω denotes the volume element on the manifold $\mathcal{D}$, i.e. $\Omega = \mathrm{d}X^1 \wedge \cdots \wedge \mathrm{d}X^d$ with $\dim(\mathcal{D}) = d$ and $\Omega_A = \partial_A \rfloor \Omega$ (the boundary volume form). We restrict ourselves to first order densities, i.e. $\mathcal{F} \in C^\infty(\mathcal{J}^1(\mathcal{X}))$ is met and additionally we denote by $F = \int_{\mathcal{D}} \mathfrak{F}$ the integrated quantity, where of course the map $x = \Phi(X)$ leading to $x_A = \partial_A\Phi(X)$ has to be plugged in to be able to evaluate the integral properly.

Based on the bundle structure $\mathcal{X} \to \mathcal{D}$ let us introduce the vertical tangent bundle $\mathcal{V}(\mathcal{X})$, as well as the tensor bundle $\mathcal{W}^d_1(\mathcal{X}) = \mathcal{T}^*(\mathcal{X}) \wedge (\overset{d}{\wedge}\mathcal{T}^*(\mathcal{D}))$, see also Giachetta et al. (1997), with a typical element $\omega = \omega_\alpha \mathrm{d}x^\alpha \wedge \Omega$ for $\mathcal{W}^d_1(\mathcal{X})$ and a typical element $v = v^\alpha\partial_\alpha$ for $\mathcal{V}(\mathcal{X})$, where the functions ω_α may depend on derivative coordinates. Furthermore, when v^α depend on derivative coordinates we call v a generalized vertical vector field; see Olver (1986).

An important object is the horizontal exterior derivative d_h, which meets $\mathrm{d}_h(\phi) = \mathrm{d}X^A \wedge d_A(\phi)$ acting on a differential form ϕ, where $d_A(\phi)$ denotes the Lie-derivative of ϕ with respect to d_A (see the appendix for more details concerning the relationship of d and d_h and the Stokes theorem in that context). Furthermore, we will treat linear differential operators (of order k) that are of the following form $\mathfrak{D} : \mathcal{W}^d_1(\mathcal{X}) \to \mathcal{V}(\mathcal{X})$, that map an element $\mathcal{W}^d_1(\mathcal{X})$ of jet-order p to an element $\mathcal{V}(\mathcal{X})$ of jet-order $p+k$. In coordinates we have

$$\mathfrak{D}(\eta) = \mathfrak{D}^{\alpha\beta\bar{K}} d_{\bar{K}}(\eta_\alpha)\partial_\beta$$

with $\#\bar{K} \le k$ (unordered multi-index $\bar{K}$) and with $\eta \in \mathcal{W}^d_1(\mathcal{X})$, where we use the compact notation $d_{\bar{L}} = d_{l_r} \circ \cdots \circ d_{l_1}$ for $\#\bar{L} = r$ with $l_i = \{1, \ldots, d\}$. The (formally) adjoint operator $\mathfrak{D}^*$ follows by integration by parts and fulfills the condition

$$\mathfrak{D}(\eta)\rfloor\bar{\eta} = \mathfrak{D}^*(\bar{\eta})\rfloor\eta + \mathrm{d}_h(\mathfrak{d}) \tag{1}$$

with $\eta, \bar{\eta} \in \mathcal{W}^d_1(\mathcal{X})$, where $\mathfrak{d}$ is a bilinear expression involving the total derivatives up to order $k-1$; see Olver (1986).

Remark 1. The differential operator $\mathfrak{D}$ is called (formally) self-adjoint if $\mathfrak{D}^* = \mathfrak{D}$ and it is called (formally) skew-adjoint if $\mathfrak{D}^* = -\mathfrak{D}$; see Olver (1986). It should be noted that it is not required that the boundary term vanishes in (1). This will be important since non-zero energy flow through the boundary can take place in our setting. This is similar to the main idea used in the Stokes–Dirac approach as in Macchelli and Melchiorri (2004b), Macchelli et al. (2004a,b), Maschke and van der Schaft (2005) and van der Schaft and Maschke (2002).

3. System representation

In this section we will introduce port-Hamiltonian systems described by pdes based on a power balance relation. This means that the system is introduced in such a way that the power balance relation together with the structure of the equations represents the physical process. Before we are able to introduce the corresponding system representations we need to introduce some geometric concepts which will be exploited later on with respect to the derivation of the power balance relation.