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Brief paper

Improved results on stability of continuous-time switched positive linear systems[☆]

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ABSTRACT

In this paper, the problems of stability for switched positive linear systems (SPLSs) under arbitrary switching are investigated in a continuous-time context. The so-called "copositive polynomial Lyapunov function" (CPLF) giving a generalization of copositive types of Lyapunov function is first proposed, which is formulated in a higher order form of the positive states of the underlying systems. It is illustrated in this paper that some classical types of Lyapunov functions can be seen as special cases of the proposed CPLF. Then, new stability conditions are developed by the new Lyapunov function approach. It is also proved that the conservativeness of the obtained criteria can be further reduced as the degree of the Lyapunov function increases. A numerical example is given to demonstrate the effectiveness and less conservativeness of the developed techniques.

1. Introduction

Switched systems are hybrid systems with both continuous dynamics and discrete events (Sun, Liu, Rees, & Wang, 2008). During the past decades, considerable attention has been devoted to the investigation of such systems due to the fact that switched systems provide a unified framework for mathematical modeling of many practical systems (Sun & Ge, 2004).

Among these studies, stability analysis is a very crucial and fundamental problem in the area of switched systems (Vu, Chatterjee, & Liberzon, 2007; Vu & Liberzon, 2008; Zhao, Liu, & Zhang, 2013; Zhao, Zhang, Shi, & Liu, 2012). When the switching mechanism is unknown and unmeasurable, the stability issue is thus the guaranteed stability under arbitrary switching (Sun, Zhao, & Hill, 2006; Zhao & Hill, 2008). For switched linear systems under arbitrary switching, it has been proved that global asymptotic stability is equivalent to the existence of a common Lyapunov function that is positive definite, smooth, and strictly decreasing along any state trajectory of each subsystem. Due to this important relationship between asymptotic stability and common Lyapunov functions, a number of efforts have been paid to find various common Lyapunov functions for different classes of switched linear systems, e.g. common quadratic Lyapunov functions (Wang, Wang, & Shi, 2009a,b) and recently common homogeneous polynomial Lyapunov functions (Chesi, Colaneri, Geromel, Middleton, & Shorten, 2012).

As a special class of switched linear systems, switched positive linear systems (SPLSs) have also been extensively studied due to their numerous applications in many fields such as communication systems, networks employing transmission control protocols and formation flying, etc. Readers may refer to Hernandez-Vargas, Colaneri, Middleton, and Blanchini (2011) and the references therein for other applications of such systems. Different from general switched systems, the states of SPLS are always confined to the positive orthant, rather than the whole state space. The positivity will bring some interesting and special properties to SPLSs (Liu, 2009). For example, time delay may lead to instability of a switched system, but for SPLSs, the stability is independent of delays (Liu & Dang, 2011; Liu, Yu, & Wang, 2010). Therefore, many challenging control issues have arisen for SPLSs, attracting much attention, particularly the stability analysis.

In stability analysis of SPLSs under arbitrary switching, it should be pointed out that, those common Lyapunov function approaches developed for general switched systems are still applicable for SPLSs (Alonso & Rocha, 2010). However, the resulting stability conditions are generally conservative for SPLSs, since the states of a SPLS are naturally positive (at least nonnegative). On the other hand, the copositive Lyapunov function approach has recently

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been proposed for SPLSs. In particular, the so-called common linear copositive Lyapunov function (CLCLF) approach has been shown to be quite efficient in stability analysis of SPLSs subject to arbitrary switching. It is conjectured by the authors of Mason and Shorten (2003) that the existence of a CLCLF for a given SPLS can be determined by testing the Hurwitz-stability of an associated convex set of matrices. Then, in Gurvits, Shorten, and Mason (2007), the conjecture is verified to be true in some specified cases, but false in general. In recent years, the existence of a CLCLF has been independently investigated in detail (Fornasini & Valcher, 2010; Knorn, Mason, & Shorten, 2009; Mason & Shorten, 2007a,b), leading to deeper insights into the properties the subsystems family must satisfy. Recently, the concept of the common quadratic copositive Lyapunov function (CQCLF) has been proposed in Fornasini and Valcher (2010) for SPLSs. However, how to numerically construct a CQCLF is still an open and interesting problem. Meanwhile, some other control synthesis results based on copositive Lyapunov function approaches, such as stabilization, H_∞ control and filtering, etc., have also been reported (Li, Lam, & Shu, 2010; Li, Lam, Wang, & Date, 2011; Shu, Lam, Gao, Du, & Wu, 2008).

However, it is noted that the copositive Lyapunov functions in the literature are commonly formulated in positive states of degree-1. From a mathematical point, finding a low order polynomial to approximate the desired Lyapunov function unavoidably leads to some conservativeness over higher order polynomials, which has already been verified to be true for general switched systems (Chesi et al., 2012). Here, an interesting question naturally arises: Can some existing results be improved by generalizing the CLCLF to some higher order copositive Lyapunov functions?

In summary, there are still many open problems and questions relating to the stability of SPLSs, some of which will be discussed towards the end of this paper.

This paper aims to explore less conservative stability conditions for switched positive linear systems under arbitrary switching. One of the major contributions of this paper is to generalize the common copositive Lyapunov function to the so-called copositive polynomial Lyapunov function (CPLF) with any specified order. Based on the new type of Lyapunov function, relaxed stability conditions are also derived and formulated in terms of a set of linear matrix inequalities. The remainder of the paper is organized as follows. Section 2 reviews necessary definitions and lemmas for SPLSs. In Section 3, the CPLF is first proposed, and it will be shown that a CPLF is less conservative than other available Lyapunov functions for SPLSs. Sufficient conditions for the existence of a CPLF ensuring the stability of a given SPLS will be derived, upon which, some corollaries are also given. Then, the relationships among these conditions are discussed as well. A numerical example is used in Section 4 to demonstrate the validity and less conservativeness of the obtained results. Finally, conclusions are addressed in Section 5.

Notations: In this paper, the notations used are fairly standard. $x \succ 0$ (or $x \succeq 0$) means that x is positive (or nonnegative); the notation $P > 0$ (≥ 0) means that P is a real symmetric and positive definite (semi-positive definite) matrix; $\mathbb{R}$, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the field of real numbers, n-dimensional Euclidean space and the space of $n \times n$ matrices with real entries, respectively, and $\mathbb{R}_0^n$ denotes $\mathbb{R}^n \setminus \{0^n\}$; $\mathbb{R}_+^n$ stands for the nonnegative orthant in $\mathbb{R}_0^n$; $\mathbb{N}_+$ stands for the set of positive integers; I_n is the identity matrix of order n; $\mathcal{I}_m \in \mathbb{R}^{(n-m+1)\times n}$ is a matrix composed of the rows $m, m+1, \ldots, n$ of I_n; $x^{[m]} := [x_1^m \quad x_2^m \quad \cdots \quad x_n^m]^T$; $\sqrt{x} = x^{[\frac{1}{2}]}$; for vectors x, y, $[x; y] = [x^T, y^T]^T$; $x^{\{m\}} := [x^{[m]}; x_1^{m-1}(\mathcal{I}_2 x); \ldots; x_1(\mathcal{I}_2 x)^{\{m-1\}}; \ldots; x_{n-1}^{m-1}(\mathcal{I}_n x); \ldots; x_{n-1}(\mathcal{I}_n x)^{\{m-1\}}]$ is a base vector containing all homogeneous monomials of degree m in x; $\lambda(A)$ are the eigenvalues of A; $A \equiv [a_{ij}]_{n\times n}$, where a_{ij} is the ith line and jth column entry of A; $\vartheta(n, m) \equiv (n+m-1)!/((n-1)!m!)$; $A \otimes B$ refers to the Kronecker product of matrices A and B; $x^{\otimes m} = \overbrace{x \otimes x \otimes \cdots \otimes x}^{m}$ denotes the mth Kronecker power in x; in addition κ_{ij} represents the Kronecker delta function.

2. Problem statements and preliminaries

Consider the following switched linear system comprising a set of positive subsystems:

$$\dot{x}(t) = A_{\sigma(t)}x(t), x(0) \succeq 0 \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $\sigma(t)$ is an arbitrary switching signal, taking its values in the finite set $S = \{1, \ldots, M\}$, *andM* is the number of subsystems. When $\sigma(t) = p \in S$, we say the pth subsystem of (1) is activated. Moreover, the system matrix $A_p, \forall p \in S$ is a Metzler matrix that is defined below.

Definition 1 (*Knorn et al., 2009*)**.** If all the off-diagonal entries of the matrix A are nonnegative, then, A is called a Metzler matrix.

Definition 2 (*Fornasini & Valcher, 2010*)**.** A linear system $\dot{x}(t) = Ax(t)$ is said to be positive if $x(0) \in \mathbb{R}_+^n$ implies that $x(t) \in \mathbb{R}_+^n$ for all $t \geq 0$.

Lemma 1 (*Fornasini & Valcher, 2010*)**.** *A linear system $\dot{x}(t) = Ax(t)$ is positive if and only if A is a Metzler matrix.*

Remark 1. Suppose that $A_p, \forall p \in S$, are Metzler matrices. We can see from Lemma 1 that all subsystems of system (1) are positive. This together with Definition 2 implies that the states of system (1) with nonnegative initial conditions will stay in the positive orthant during the running time of the first operation mode. Therefore, the initial conditions of the next activated subsystem are also nonnegative, which in turn means the state trajectory of system (1) under arbitrary switching will always stay in the positive orthant. In the literature, system (1) is commonly termed as a switched positive linear system (SPLS).

On the basis of Lyapunov stability theory, this paper aims to explore less conservative stability conditions for SPLS (1). Before ending this section, the following preliminaries are given, which will be used to develop our main results later.

Lemma 2 (*LaSalle, 1961*)**.** *Let $x^* = 0$ be an equilibrium of the autonomous system $\dot{x}(t) = f(x(t))$, if $V(x(t)) : \mathbb{R}^n \to \mathbb{R}$ is a continuous scalar function satisfying the following conditions:*

(I) $V(0) = 0$;
(II) *$V(x(t))$ is a continuously differentiable function in Ω;*
(III) $V(x(t)) > 0, \forall x(t) \in \Omega \setminus \{0\}$;
(IV) $\dot{V}(x(t)) < 0, \forall x(t) \in \Omega \setminus \{0\}$;

where Ω represents a state region, then $V(x(t))$ is a Lyapunov function to prove asymptotic stability of the underlying system in Ω.

Remark 2. For SPLS (1), the state region Ω to be considered is the positive orthant. Also note from Remark 1 that the considered Ω is positively invariant when $A_p, \forall p \in S$, are Metzler matrices.

Lemma 3 (*Fornasini & Valcher, 2010*)**.** *The linear function $V(x) = x^T v$ defines a common linear copositive Lyapunov function (CLCLF) for SPLS (1), if and only if the vector v satisfies:*

(I) $v \succ 0$;
(II) $A_p^T v \prec 0, \forall p \in S$.