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Attitude synchronization control for a group of flexible spacecraft[☆]

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ABSTRACT

To solve the problem of attitude synchronization for a group of flexible spacecraft during formation maneuvers, a distributed attitude cooperative control strategy is investigated in this paper. Based on the backstepping design and the neighbor-based design rule, a distributed attitude control law is constructed step by step. Using cascaded systems' theory and graph theory, it is shown that the attitude synchronization is achieved asymptotically and the induced vibrations by flexible appendages are simultaneously suppressed under the proposed control law.

1. Introduction

Distributed cooperative control of multi-agent systems has been attracting a lot of interest in control community recently because of its many advantages, such as greater efficiency, higher robustness, and less communication requirement (Hong, Hu, & Gao, 2006; Khoo, Xie, & Man, 2009; Ren & Beard, 2007). As an important application area of distributed control, the attitude cooperative control for spacecraft formation has also gained certain progresses.

For a group of rigid spacecraft, in Lawton and Beard (2002), two kinds of distributed control strategies were designed such that the attitude synchronization is achieved under a ring communication graph. Later, this ring communication topology graph was relaxed to be a more general case in Ren (2007). When the angular velocity is unmeasurable, the attitude synchronization control problem was also investigated in Abdessameud and Tayebi (2009) and Lawton and Beard (2002). For the attitude cooperative tracking control problem with a single leader or multiple leaders, the distributed cooperative control laws were proposed in Dimarogonas, Tsiotras, and Kyriakopoulos (2009) and Wu, Wang, and Poh (2013). Recently, in order to enhance the convergence rate, precision, and robustness against disturbances, the finite-time control technique (Bhat & Bernstein, 2000; Qian & Li, 2005; Shen & Xia, 2008) has been employed to design finite-time attitude synchronization control algorithms (Du & Li, 2011; Li, Du, & Lin, 2011; Meng, Ren, & You, 2010).

Note that all the preceding listed studies on attitude cooperative control only concentrate on the rigid spacecraft. Nevertheless, with the development of the space science technology, the structure of spacecraft will be more complex and usually carry some flexible appendages, such as solar array, manipulator, etc. Compared with the rigid spacecraft, the control problem of flexible spacecraft becomes more complicated since not only the attitude control but also the vibration induced by the flexible appendages is required to be handled, where the coupling nonlinearities with modal variable are the main obstructions. Although for a single flexible spacecraft, many researchers have developed different nonlinear control methods, such as Gennaro (2003) and Hu (2010), to name just a few. However, for the attitude cooperative control for multiple flexible spacecraft, to the best of our knowledge, there have been no available results.

In this paper, we focus on solving the problem of attitude synchronization for a group of flexible spacecraft. Based on the backstepping design, a distributed attitude cooperative control law is explicitly constructed in two steps. At the first step, the angular velocity is regarded as a virtual control input and a neighbor-based distributed control law is designed, where the modal variables are first assumed to be measurable. Then to address the problem of lack of modal variables measurement, the virtual controller is redesigned together with a modal observer. At the second step, for the dynamic subsystem, a control law is designed for the control torque such that the virtual angular velocity can be tracked by the real velocity. Finally, a rigorous stability analysis for the overall closed-loop system is given based on cascaded systems' theory.

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2. Preliminaries and problem formulation

2.1. Graph theory

Without loss of generality, n flexible spacecraft will be considered in this paper. Let $\Gamma = \{1, \ldots, n\}$. Each spacecraft is regarded as a node and the information exchange among n spacecraft is denoted by a directed graph $G(A) = \{V, E, A\}$. $V = \{v_i, i = 1, \ldots, n\}$ is the set of nodes, $E \subseteq V \times V$ is the set of edges and $A = [a_{ij}] \in R^{n\times n}$ is the weighted adjacency matrix of the graph $G(A)$ with nonnegative adjacency elements a_{ij}. If there is an edge from node j to node i, i.e., $(v_j, v_i) \in E$, then $a_{ij} > 0$, which means there exists an available information channel from node j to node i. Moreover, we assume that $a_{ii} = 0$ for all $i \in \Gamma$. The set of neighbors of node i is denoted by $N_i = \{j : (v_j, v_i) \in E\}$. The out-degree of node v_i is defined as $\deg_{out}(v_i) = d_i = \sum_{j=1}^{n} a_{ij} = \sum_{j\in N_i} a_{ij}$. Then the degree matrix of digraph G is $D = \mathrm{diag}\{d_1, \ldots, d_n\}$ and the Laplacian matrix of digraph G is $L = D - A$.

A path in directed graph G from v_{i_1} to v_{i_k} is a sequence of $v_{i_1}, v_{i_2}, \ldots, v_{i_k}$ of finite nodes starting with v_{i_1} and ending with v_{i_k} such that $(v_{i_l}, v_{i_{l+1}}) \in E$ for $l = 1, 2, \ldots, k-1$. The directed graph G is strongly connected if there is a path between any two distinct vertices.

2.2. Flexible spacecraft attitude model

The model of flexible spacecraft attitude consists of two parts: the kinematic model and the dynamic model. Based on the quaternion (Shuster, 1993), the kinematic equation of the ith spacecraft is described by

$$\dot{q}_i = \frac{1}{2}E(q_i)\omega_i, \quad i \in \Gamma = \{1, \ldots, n\}, \tag{1}$$

where $q_i = [q_{i,0}, q_{i,1}, q_{i,2}, q_{i,3}]^T = [q_{i,0}, q_{i,v}^T]^T$ is unit quaternion, $\omega_i = [\omega_{i,1}, \omega_{i,2}, \omega_{i,3}]^T$ is the angular velocity vector, and $E(q_i) = \begin{pmatrix} -q_{i,v}^T \\ -s(q_{i,v}) + q_{i,0}I_3 \end{pmatrix}$, where I_3 denotes the 3×3 identity matrix and $s(\cdot)$ denotes the skew matrix. The skew matrix is defined as $s(x) = \begin{bmatrix} 0 & x_3 & -x_2 \\ -x_3 & 0 & x_1 \\ x_2 & -x_1 & 0 \end{bmatrix}$ for any $x = [x_1, x_2, x_3]^T \in R^3$, which satisfies $\|s(x)\| = \|x\|$. In addition, the unit quaternion satisfies the constraint condition

$$q_{i,0}^2 + q_{i,v}^T q_{i,v} = 1. \tag{2}$$

From Gennaro (2003), the dynamic equation of the ith spacecraft is

$$\begin{aligned} &J_i\dot{\omega}_i + \delta_i^T\ddot{\eta}_i = s(\omega_i)(J_i\omega_i + \delta_i^T\dot{\eta}_i) + \tau_i, \\ &\ddot{\eta}_i + C_i\dot{\eta}_i + K_i\eta_i = -\delta_i\dot{\omega}_i \quad i \in \Gamma, \end{aligned} \tag{3}$$

where $J_i = J_i^T$ is the positive definite inertia matrix, $\tau_i = [\tau_{i,1}, \tau_{i,2}, \tau_{i,3}]^T$ is the control torque vector, δ_i is the coupling matrix between the rigid body and the flexible attachments, η_i is the vector of the modal coordinate, $C_i = \mathrm{diag}\{2\xi_{i,j}\omega_{i,nj}, j = 1, \ldots, N_i\}$ is the damping (diagonal) matrix, $K_i = \mathrm{diag}\{\omega_{i,nj}, j = 1, \ldots, N_i\}$ is the stiffness matrix, N_i is the number of flexible attachments for ith spacecraft, $\omega_{i,nj}$ is the natural frequencies and $\xi_{i,j}$ is the associated damping.

As that in Gennaro (2003), denote $\psi_i = \dot{\eta}_i + \delta_i\omega_i$ and $J_{m,i} = J_i - \delta_i^T\delta_i$. The attitude Eqs. (1) and (3) can be rewritten as

$$\begin{aligned} &\dot{q}_i = \frac{1}{2}E(q_i)\omega_i, \qquad \dot{\eta}_i = \psi_i - \delta_i\omega_i, \\ &\dot{\psi}_i = -(C_i\psi_i + K_i\eta_i - C_i\delta_i\omega_i), \\ &J_{m,i}\dot{\omega}_i = s(\omega_i)(J_{m,i}\omega_i + \delta_i^T\psi_i) \\ &\qquad + \delta_i^T(C_i\psi_i + K_i\eta_i - C_i\delta_i\omega_i) + \tau_i, \quad i \in \Gamma. \end{aligned} \tag{4}$$

2.3. Control objective

The goal of this paper is to design a distributed attitude control law for the n flexible spacecraft such that all the attitudes can reach consensus/synchronization and the induced oscillations of the spacecraft flexible appendages are damped out.

Lemma 1 (Xiao, Wang, Chen, & Gao, 2009). *If a directed graph G is strongly connected, then there is a positive vector $\gamma = [\gamma_1, \ldots, \gamma_n]^T \in R^n$ (i.e. $\gamma_i > 0, i = 1, \ldots, n$) such that $\gamma^T L = 0$, where L the corresponding Laplacian matrix L of graph G.*

3. Main results

The controller design method is mainly based on the backstepping design. Specifically speaking, the design procedure is divided into two steps:

(i) For the kinematic subsystem and modal dynamics

$$\begin{aligned} &\dot{q}_i = \frac{1}{2}E(q_i)\omega_i, \qquad \dot{\eta}_i = \psi_i - \delta_i\omega_i, \\ &\dot{\psi}_i = -(C_i\psi_i + K_i\eta_i - C_i\delta_i\omega_i), \quad i \in \Gamma, \end{aligned} \tag{5}$$

considering ω_i as the virtual input, a virtual angular velocity ω_i^* is designed such that the attitudes of the kinematic subsystem achieve consensus. Here, we first consider the case that the modal variables η_i and ψ_i are measured and then consider the unmeasured case.

(ii) For the dynamic subsystem, a control law τ_i is designed such that the virtual velocity can be tracked by the real angular velocity.

3.1. Virtual angular velocity design

In this subsection, the angular velocity ω_i is regarded as a virtual control input and is designed such that the attitude synchronization can be achieved. We first consider the case that the modal variables η_i and ψ_i are known.

A. Case of known modal variables η_i and ψ_i

Lemma 2. *For the subsystem (5), if the directed graph $G(A)$ is strongly connected and the virtual angular velocity is designed as*

$$\begin{aligned} \omega_i^* = &-k_1 \sum_{j\in N_i} a_{ij}\Big[(q_{i,v} - q_{j,v}) + [(\psi_i^T C_i - 2\eta_i^T K_i)\delta_i]^T \\ &- [(\psi_j^T C_j - 2\eta_j^T K_j)\delta_j]^T\Big], \quad i \in \Gamma, \end{aligned} \tag{6}$$

where $k_1 > 0$, then the attitude synchronization can be achieved asymptotically.

Proof. According to Lemma 1, if the directed graph $G(A)$ is strongly connected, there exists a positive column vector $\gamma = [\gamma_1, \ldots, \gamma_n]^T \in R^n$ such that $\gamma^T L = 0$. Consider the following candidate Lyapunov function

$$\begin{aligned} V_1 = \sum_{i=1}^{n}\gamma_i W_i, \ W_i = \Big[&(2 - 2q_{i,0}) + \frac{1}{2}\psi_i^T\psi_i + \eta_i^T K_i\eta_i \\ &+ \frac{1}{2}(\psi_i + C_i\eta_i)^T(\psi_i + C_i\eta_i)\Big]. \end{aligned} \tag{7}$$

By the definition of $E(q_i)$, the derivative of W_i along system (5) is

$$\dot{W}_i = -\eta_i^T C_i K_i\eta_i - \psi_i^T C_i\psi_i + [q_{i,v}^T + (\psi_i^T C_i - 2\eta_i^T K_i)\delta_i]\omega_i. \tag{8}$$

Denote

$$\beta_i = q_{i,v} + [(\psi_i^T C_i - 2\eta_i^T K_i)\delta_i]^T, \tag{9}$$