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Identification of microwave filters by analytic and rational H^2 approximation[☆]

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ABSTRACT

In this paper, an original approach to frequency identification is explained and demonstrated through an application in the domain of microwave filters. This approach splits into two stages: a stable and causal model of high degree is first computed from the data (completion stage); then, model reduction is performed to get a rational low order model. In the first stage the most is made of the data taking into account the expected behavior of the filter. A reduced order model is then computed by rational H^2 approximation. A new and efficient method has been developed, improved over the years and implemented to solve this problem. It heavily relies on the underlying Hilbert space structure and on a nice parameterization of the optimization set. This approach guarantees the stability of the MIMO approximant of prescribed McMillan degree.

1. Introduction

The microwave filters that we consider are used in telecommunication satellites for channel multiplexing.

These electromagnetic waveguide filters are made of resonant cavities (see Fig. 1) interconnected by coupling irises (orthogonal double slits). Each cavity has 3 screws which allow one to tune the filter. Using a low-pass transformation these high-pass filters are usually modeled by a low-pass electrical circuit (see Fig. 2). In this model, Ω is the normalized frequency, each resonant cavity mode is represented by a fictive resonant circuit (frequency M_{kk}) and the coupling between modes (produced by the irises) by impedance inverters (jM_{kl}). In the remainder of the paper we will adopt the mathematical notation, i rather than j, for the square root of -1. Electrical power transfer is then described by a scattering matrix. From a mathematical viewpoint, the scattering matrix R is a rational matrix function with *complex coefficients* which is stable (poles with negative real parts) lossless ($R(i\omega)$ is unitary) and symmetric. The geometry of the filter is characterized by the electrical parameters which appear on a realization in particular form of the scattering matrix. Namely, $R(s) = I + C(sI - A)^{-1}B$ with

$$C = \begin{bmatrix} i\sqrt{2r_1} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & i\sqrt{2r_2} \end{bmatrix}, \qquad B = C^t,$$
$$A = -R - iM, \qquad A = A^t \qquad r = -\frac{1}{2}C^tC, \tag{1}$$

where r_1 and r_2 are the input and output loads and the matrix M is the *coupling matrix*. The structure of M (non-zeros entries) specifies the way resonators are coupled to one another. The McMillan degree of R corresponds to the number of circuits, that is the number of resonant modes or else two times the number of cavities.

The problem of extracting coupling parameters from frequency scattering measurements is essential with a view to reducing the cost of hardware and CAD tuning. The direct approach consists in feeding to a generic optimizer the function evaluating the scattering matrix from the coupling parameters, in order to fit the data. However, it often depends on a favorable initial guess and substantial efforts are currently being spent to design more robust methods. Another approach consists in first identifying a rational (linear) model from the data. Then, the coupling parameters are extracted from this rational model using classical design methods.

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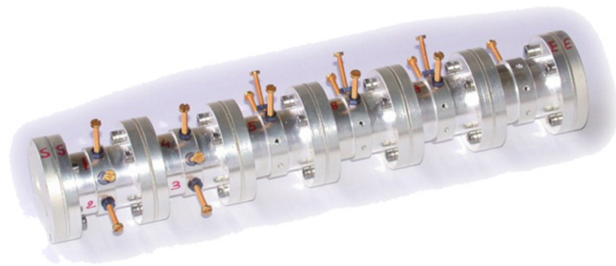

Fig. 1. A microwave filter.

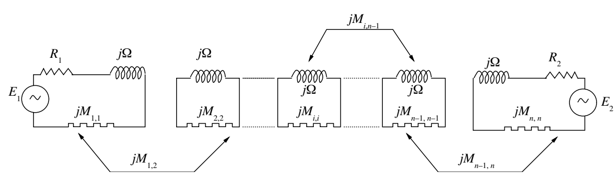

Fig. 2. Low-pass prototype.

In the filter community, the so-called Cauchy method is widely used to compute the rational model (Adve, Sarkar, Rao, Miller, & Pflug, 1997; Lampérez, Sarkar, & Palma, 2002). Let us point out three major problems encountered in this direction, and in many other methods proposed in the literature:

- there is no guarantee on the stability of the rational model, i.e. the derived model can have unstable poles;
- there is no control on the McMillan degree of the model;
- no constraint is imposed to the model outside the frequency band of measurement (broadband), which may result in unrealistic behavior there.

Many toolboxes propose input–output identification, while a few deal with frequency data. The software Vector Fitting and its Matrix Fitting extension (see Gustavsen & Semlyen, 1999, 2004 and the bibliography therein) has become popular in the electromagnetic simulation community. However, the convergence towards a stable rational approximant, optimal in some least-square sense, is not guaranteed by this algorithm. Moreover no control is given in the MIMO case on the overall McMillan degree of the result, but only on its number of distinct poles. The same problem arises with the Frequency Domain Identification Toolbox (Kollár, 2004) which only deals with SISO systems. This is unacceptable for the application we have in mind, in which the target McMillan degree is prescribed in advance and given by the number of coupled resonators present in the equivalent circuit of the filter.

To overcome these difficulties, we have developed a two stage approach to identify a rational model from the scattering data. A stable and causal model of high degree is first computed from the data (completion stage); then, model reduction is performed to get a model of the prescribed order. The first stage will be addressed in Section 3. Then we will consider the model reduction step. We tackle this problem using a rational H^2 approximation and the original approach developed over the years in Baratchart, Cardelli, and Olivi (1991), Fulcheri and Olivi (1998) and Marmorat, Olivi, Hanzon, and Peeters (2002). We present here the state of the art of this approach which includes an efficient parameterization of balanced output pairs. The exposition is definitely application oriented, so that the emphasis will be put on the effective implementation of the method.

2. The Hilbert space framework

To deal with these completion and model reduction problems, we thus favor an approach based on approximation. A relevant context to deal with approximation is that of a Hilbert space. On the other hand, stability and causality of a rational model are equivalent to the analyticity of the transfer function in the closed right half-plane (poles at finite distance in the open left half-plane). We denote by $\mathbb{C}^+$ and $\mathbb{C}^-$ the open right and left half-planes. To properly handle stability and causality, we embed rational functions in a larger space of analytic functions in $\mathbb{C}^+$, namely a Hardy space naturally endowed with an L^2 norm. Note that, due to the low-pass transformation, the frequency data and the model that we consider do not satisfy the conjugacy requirement. This is why we consider Hardy spaces of *complex* functions.

The usual Hardy space of the half-plane, $H^2(\mathbb{C}^+)$, consists of functions f analytic in $\mathbb{C}^+$, whose L^2-norm remains uniformly bounded on vertical lines,

$$\sup_{x>0} \int_{-\infty}^{\infty} |f(x+i\omega)|^2 d\omega < \infty.$$

The Hardy space of the left half-plane, $H^2(\mathbb{C}^-)$, is defined in a similar way. An important fact is that the Laplace transform gives an isometry from $L^2(\mathbb{R}^\pm)$ onto $H^2(\mathbb{C}^\pm)$. It allows one to consider these Hardy spaces as subspaces of $L^2(i\mathbb{R})$, the image of $L^2(\mathbb{R})$ by the Laplace transform (Partington, 1997). Moreover,

$$L^2(i\mathbb{R}) = H^2(\mathbb{C}^+) \oplus H^2(\mathbb{C}^-).$$

Each function in $L^2(i\mathbb{R})$ can thus be decomposed as the sum of a function in $H^2(\mathbb{C}^+)$ (stable part) and a function in $H^2(\mathbb{C}^-)$ (anti-stable part).

However, a stable causal function which fails to be strictly proper (to be 0 at ∞) does not belong to $L^2(i\mathbb{R})$. In order to include these functions in our setting, we shall replace the usual Lebesgue measure by the weighted measure $d\mu(w) = \frac{d\omega}{1+\omega^2}$, which also has the advantage to penalize high frequencies. The associated Hardy spaces $H^2_\mu(\mathbb{C}^+)$ and $H^2_\mu(\mathbb{C}^-)$ are defined in a similar way and can be viewed as subspaces of the space $L^2(d\mu)$ of functions defined on the imaginary axis and such that

$$\|f\|_\mu^2 = \int_{-\infty}^{\infty} |f(i\omega)|^2 \frac{d\omega}{1+\omega^2} < \infty.$$

However, $H^2_\mu(\mathbb{C}^+)$ and $H^2_\mu(\mathbb{C}^-)$ fail to be orthogonal complements, since their intersection is not empty (it contains for example constant functions). For $f \in L^2(d\mu)$, we denote by $P_+(f)$ its orthogonal projection onto $H^2_\mu(\mathbb{C}^+)$ (stable part) and by $P_-(f)$ its orthogonal projection on the orthogonal complement of $H^2_\mu(\mathbb{C}^+)$ (unstable part). Hardy spaces thus provide an interesting tool to estimate causality and stability of a given transfer function.

3. Compensation of delay components and completion of the data

After the low-pass frequency transformation, we suppose that the harmonic scattering measurements of the filter yield the knowledge of a 2×2 matrix function $\tilde{S}(iw)$ defined on a strict sub-interval J of the imaginary axis. In practice this function is obtained thanks to the interpolation (splines) of a discrete set of measurement points. The mathematical model we want to identify from these measurements is given by

$$\begin{bmatrix} e^{i\frac{\alpha}{2}h(w)} & 0 \\ 0 & e^{i\frac{\beta}{2}h(w)} \end{bmatrix} R(i\,w) \begin{bmatrix} e^{i\frac{\alpha}{2}h(w)} & 0 \\ 0 & e^{i\frac{\beta}{2}h(w)} \end{bmatrix},$$

where R is the 2×2 *rational* scattering matrix of the low-pass model of the filter and the exponential terms are due to the access lines used to perform the measurements. The transformation $h(w)$ maps normalized frequencies (low-pass model) to high frequencies (original system).