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Optimal structured static state-feedback control design with limited model information for fully-actuated systems[☆]

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ABSTRACT

We introduce the family of limited model information control design methods, which construct controllers by accessing the plant's model in a constrained way, according to a given design graph. We investigate the closed-loop performance achievable by such control design methods for fully-actuated discrete-time linear time-invariant systems, under a separable quadratic cost. We restrict our study to control design methods which produce structured static state feedback controllers, where each subcontroller can at least access the state measurements of those subsystems that affect its corresponding subsystem. We compute the optimal control design strategy (in terms of the competitive ratio and domination metrics) when the control designer has access to the local model information and the global interconnection structure of the plant-to-be-controlled. Finally, we study the trade-off between the amount of model information exploited by a control design method and the best closed-loop performance (in terms of the competitive ratio) of controllers it can produce.

1. Introduction

Many modern control systems, such as aircraft and satellite formation (Giulietti, Pollini, & Innocenti, 2000; Kapila, Sparks, Buffington, & Yan, 1999), automated highways and other shared infrastructure (Negenborn, Lukszo, & Hellendoorn, 2010; Swaroop & Hedrick, 1999), flexible structures (Joshi, 1989), and supply chains (Braun, Rivera, Flores, Carlyle, & Kempf, 2003; Dunbar, 2007), consist of a large number of subsystems coupled through their performance goals or system dynamics. When regulating this kind of plant, it is often advantageous to adopt a distributed control architecture, in which the controller itself is composed of interconnected subcontrollers, each of which accesses a strict subset of the plant's output. Several control synthesis methods have been proposed over the past decades that result in distributed controllers of this form, with various types of closed-loop stability and performance guarantees (e.g., Ayres de Castro and Paganini (1999), Bamieh, Paganini, and Dahleh (2002), Chen and Lall (2003), Hu (1994), Levine, Johnson, and Athans (1971), Scorletti and Duc (2001), Söderström (1978), Wang and Davison (1973) and Wenk and Knapp (1980)). Most recently, the tools presented in Rotkowitz and Lall (2006) and Voulgaris (2003) revealed how to exploit the specific interconnection of classes of plants (the so-called quadratically invariant systems) to formulate convex optimization problems for the design of structured H_∞- and H_2-optimal controllers. A common thread in this part of the literature is the assumption that, even though the controller is structured, its design can be performed in a centralized fashion, with full knowledge of the plant model. However, in some applications (described in more detail in the next paragraph), this assumption is not always warranted, as the design of each subcontroller may need to be carried out by a different control designer, with no access to the global model of the plant, although its interconnection structure and the common closed-loop cost function to be minimized are public knowledge. This class of problems, which we refer to as "limited model information control design problems", is the main object of interest in the present paper.

Limited model information control design occurs naturally in contexts where the subsystems belong to different entities, which may consider their model information private and may thus be reluctant to share it with others. In this case, the designers may have to resort to "communication-less" strategies in which subcontroller K_i depends solely on the description of subsystem i's

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model. This case is well illustrated by supply chains, where the economic incentives of competing companies might limit the exchange of model information (such as, inventory volume, transportation efficiency, raw material sources, and decision process) inside a layer of the chain (see Braun et al. (2003), Lee, Padmanabhan, and Whang (1997), Riddalls, Bennett, and Tipi (2000) and Sarimveis, Patrinos, Tarantilis, and Kiranoudis (2008) for a detailed review of modeling and control of supply chains). Another reason for using communication-less strategies in more general design situations, even when the circulation of plant information is not restricted a priori, is that the resulting subcontroller K_i does not need to be modified if the characteristics of a particular subsystem, which is not directly connected to subsystem i, vary. For instance, consider a chemical plant in the process industry, with thousands of local controllers. In such a large-scale system, the tuning of each local controller should not require model parameters from other parts of the system so as to simplify maintenance and limit controller complexity. Note that engineers often implement these large-scale systems as a whole using commercially available pre-designed modules. These modules are designed, in advance, with no prior knowledge of their possible use or future operating condition. This lack of availability of the complete model of the plant, at the time of the design, constrains the designer to only use its own model parameters in each module's control design.

Control design based on uncertain plant model information is a classic topic in the robust control literature (Ball & Cohen, 1987; Doyle, 1982; Zames, 1981; Zhou & Doyle, 1998). However, designing an optimal controller without a global model is different from a robust control problem. In optimal control design with limited model information, subsystems do not have any prior information about the other subsystems' model; i.e., there is no nominal model for the design procedure and there is no bound on the model uncertainties. There have been some interesting approaches for tackling this problem. For instance, Ando and Fisher (1963); Gajtsgori and Pervozvanski (1979); Sethi and Zhang (1998); Sezer and Šiljak (1986) introduced methods for designing sub-optimal decentralized controllers without a global dynamical model of the system. In these papers, the authors assume that the large-scale system to be controlled consists of an interconnection of weakly coupled subsystems. They design an optimal controller for each subsystem using only the corresponding local model, and connect the obtained subcontrollers to construct a global controller. They show that, when coupling is negligible, this latter controller is satisfactory in terms of closed-loop stability and performance. However, as coupling strength increases, even closed-loop stability guarantees are lost. Other approaches such as Dunbar (2007) and Negenborn et al. (2010) are based on receding horizon control and use decomposition methods to solve each step's optimization problem in a decentralized manner with only limited information exchange between subsystems. What is missing from the literature, however, is a rigorous characterization of the best closed-loop performance that can be attained through limited model information design and a study of the trade off between the closed-loop performance and the amount of exchanged information. We tackle this question in the present paper for a particular class of systems (namely, the set of fully-actuated discrete-time linear time-invariant dynamical systems) and a particular class of control laws (namely, the set of structured linear static state feedback controllers where each subcontroller can at least access the state measurements of those subsystems that affect its corresponding subsystem).

In this paper, we study the properties of limited model information control design methods. We investigate the relationship between the amount of plant information available to the designers, the nature of the plant interconnection graph, and the quality (measured by the closed-loop control goal) of controllers that can be constructed using their knowledge. To do so, we look at limited model information and communication-less control design methods as belonging to a special class of maps between the plant and controller sets, and make use of the competitive ratio and domination metrics introduced in Langbort and Delvenne (2010) to characterize their intrinsic limitations. To the best of our knowledge, there are no other metrics specifically tuned to control design methods. We address much more general classes of subsystems and of limitations on the model information available to the designer than is done in Langbort and Delvenne (2010). Specifically, we consider limited model information structured static state-feedback control design for interconnections of fully-actuated (i.e., with invertible B-matrix) discrete-time linear time-invariant subsystems with quadratic separable (i.e., with block diagonal Q- and R-matrices) cost function. Our choice of such a cost function is motivated by our interest in applications such as power grids (Baughman, Siddiqi, & Zarnikau, 1997; Berger & Schweppe, 1989; Botterud, Ilic, & Wangensteen, 2005; Chao & Peck, 1996) and (Negenborn et al., 2010, chapters 5, 10), supply chains (Braun et al., 2003; Dunbar, 2007), and water level control (Negenborn et al., 2010, chapter 18), which have been shown to be well-modeled by dynamically-coupled but cost-decoupled interconnected systems. We show in the last section of the paper that the assumption on the B-matrix can be partially removed for the sinks (i.e., subsystems that cannot affect any other subsystem) in the plant graph.

We investigate the best closed-loop performance achievable by structured static state feedback controllers constructed by limited model information design strategies. We show that the result depends crucially on the plant graph and the control graph. In the case where the plant graph contains no sink and the control graph is a supergraph of the plant graph, we extend the fact proven in Langbort and Delvenne (2010) that the deadbeat strategy is the best communication-less control design method. However, the deadbeat control design strategy is dominated when the plant graph has sinks, and we exhibit a better, undominated, communication-less control design method, which, although having the same competitive ratio as the deadbeat control design strategy, takes advantage of the knowledge of the sinks' location to achieve a better closed-loop performance in average. We characterize the amount of model information needed to achieve better competitive ratio than the deadbeat control design strategy. This amount of information is expressed in terms of properties of the design graph; a directed graph which indicates the dependency of each subsystem's controller on different parts of the global dynamical model.

This paper is organized as follows. After formulating the problem of interest and defining the performance metrics in Section 2, we characterize the best communication-less control design method according to both competitive ratio and domination metrics in Section 3. In Section 4, we show that achieving a strictly better competitive ratio than these control design methods requires a complete design graph when the plant graph is itself complete. Finally, we end with a discussion on extensions in Section 5 and the conclusions in Section 6.

1.1. Notation

Sets will be denoted by calligraphic letters, such as $\mathcal{P}$ and $\mathcal{A}$. If $\mathcal{A}$ is a subset of $\mathcal{M}$ then $\mathcal{A}^c$ is the complement of $\mathcal{A}$ in $\mathcal{M}$, i.e., $\mathcal{M} \setminus \mathcal{A}$.

Matrices are denoted by capital roman letters such as A. A_j will denote the jth row of A. A_{ij} denotes a sub-matrix of matrix A, the dimension and the position of which will be defined in the text. The entry in the ith row and the jth column of the matrix A is a_{ij}.

Let S^n_{++} (S^n_+) be the set of symmetric positive definite (positive semidefinite) matrices in $\mathbb{R}^{n\times n}$. $A > (\geq) 0$ means that the symmetric matrix $A \in \mathbb{R}^{n\times n}$ is positive definite (positive semidefinite) and $A > (\geq) B$ means that $A - B > (\geq) 0$.