Automatica 49 (2013) 414–423

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Synchronization of discrete-time multi-agent systems on graphs using Riccati design☆

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ARTICLE INFO

Article history:
Received 15 April 2011
Received in revised form
30 May 2012
Accepted 4 October 2012
Available online 12 December 2012

Keywords:
Cooperative control systems
Synchronization
Multi-agent systems
Discrete-time synchronizing region
Decentralized control
Riccati equation

ABSTRACT

In this paper design methods are given for synchronization control of discrete-time multi-agent systems on directed communication graphs. The graph properties complicate the design of synchronization controllers due to the interplay between the eigenvalues of the graph Laplacian matrix and the required stabilizing gains. Two methods are given herein that decouple the design of the synchronizing gains from the detailed graph properties. Both are based on computation of the local control gains using Riccati design; the first is based on an H_∞ type Riccati inequality and the second on an H_2 type Riccati equation. Conditions are given for synchronization based on the relation of the graph eigenvalues to a bounded circular region in the complex plane that depends on the agent dynamics and the Riccati solution. The notion of 'synchronizing region' is used. An example shows the effectiveness of these design methods for guaranteeing synchronization in cooperative discrete-time systems.

1. Introduction

The last two decades have witnessed an increasing interest in multi-agent network cooperative systems, inspired by natural occurrence of flocking and formation forming. These systems are applied to formations of spacecrafts, unmanned aerial vehicles, mobile robots, distributed sensor networks etc. (Olfati-Saber, Fax, & Murray, 2007). Early work with networked cooperative systems in continuous and discrete time is presented in Fax and Murray (2004), Jadbabaie, Lin, and Morse (2003), Olfati-Saber and Murray (2003, 2004), Ren and Beard (2005) and Tsitsiklis (1984). These papers generally referred to consensus without a leader. By adding a leader that pins to a group of other agents one can obtain synchronization to a command trajectory using a virtual leader (Jadbabaie et al., 2003), also named pinning control (Li, Duan, Chen, & Huang, 2010; Wang & Chen, 2002). Necessary and sufficient conditions for synchronization are given by the master stability function, and the related concept of the synchronizing region, in Duan, Chen, and Huang (2009), Li et al. (2010) and Pecora and Carroll (1998). For continuous-time systems synchronization was guaranteed (Li et al., 2010; Tuna, 2008; Zhang & Lewis, 2011) using optimal state feedback derived from the continuous time Riccati equation. It was shown that, using Riccati design for the feedback gain of each node guarantees an unbounded right-half plane region in the s-plane. For discrete-time systems such general results are still lacking, though You and Xie (2011a) deals with single-input systems and undirected graph topology and You and Xie (2011b) deals with multivariable systems on digraphs. These were originally inspired by the earlier work of Elia and Mitter (2001) and Fu and Xie (2005) concerning optimal logarithmic quantizer density for stabilizing discrete time systems.

In this paper we are concerned with synchronization for agents described by linear time-invariant discrete-time dynamics. The interaction graph is directed and assumed to contain a directed spanning tree. For the needs of consensus and synchronization to a leader or control node we employ pinning control (Jadbabaie et al., 2003; Wang & Chen, 2002). The concept of synchronizing region (Duan et al., 2009; Li et al., 2010; Pecora & Carroll, 1998) is instrumental in analyzing the synchronization properties of cooperative control systems. The synchronizing region is the region in the complex plane within which the graph Laplacian

☆ This work is supported by AFOSR grant FA9550-09-1-0278, NSF grant ECCS-1128050, ARO grant W91NF-05-1-0314, and China NNSF grant 61120106011. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Valery Ugrinovskii under the direction of Editor Ian R. Petersen.

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doi:10.1016/j.automatica.2012.11.038

matrix eigenvalues must reside to guarantee synchronization. The crucial difference between systems in continuous time and discrete time is the form of the stability region. For continuous-time systems the stability region is the left half s-plane, which is unbounded by definition, and a feedback matrix can be chosen (Li et al., 2010; Zhang & Lewis, 2011) such that the synchronizing region for a matrix pencil is also unbounded. On the other hand the discrete-time stability region is the interior of the unit circle in the z-plane, which is inherently bounded. Therefore, the synchronizing regions are bounded as well. This accounts for stricter synchronizability conditions in discrete-time, such as those presented in You and Xie (2011a,b).

In the seminal paper (Li, Duan, & Chen, 2011) an algorithm is given based on an H_∞ type Riccati equation for synchronization control of linear discrete-time systems that have no poles outside the unit circle. The case of consensus without a leader is considered.

This paper extends results in You and Xie (2011b) to provide conditions for achieving synchronization of identical discrete-time state space agents on a directed communication graph structure. It extends results in Li et al. (2011) to the case of unstable agent dynamics. This paper considers synchronization to a leader dynamics. The concept of discrete synchronizing region in the z-plane is used.

The graph properties complicate the design of synchronization controllers due to the interplay between the eigenvalues of the graph Laplacian matrix and the required stabilizing gains. Two approaches to testing for synchronizability are given which decouple the graph properties from the feedback design details. Both give methods for selecting the feedback gain matrix to yield synchronization. The first result, based on an H_∞ type Riccati inequality, gives a milder condition for synchronization in terms of a circle whose radius is generally difficult to compute. The second result is in terms of a circle whose radius is easily computed from an H_2 Riccati equation solution, but gives a stricter condition. Both are shown to yield known results in the case of single-input systems on undirected graphs. Based on the given designs, results are given on convergence and robustness of the design. An example illustrates the usefulness and effectiveness of the proposed design.

2. Riccati decentralized feedback design for system synchronization

2.1. Graph properties and notation

Consider a graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ with a nonempty finite set of N vertices $\mathcal{V}=\{v_1,\ldots,v_N\}$ and a set of edges or arcs $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$. It is assumed that the graph is simple, *i.e.* there are no repeated edges or self-loops $(v_i,v_i)\notin\mathcal{E}$, $\forall i$. General directed graphs (digraphs) are considered and it is taken that information propagates through the graph along directed arcs. Denote the connectivity matrix as $E=[e_{ij}]$ with $e_{ij}>0$ if $(v_j,v_i)\in\mathcal{E}$ and $e_{ij}=0$ otherwise. Note that diagonal elements $e_{ii}=0$. The set of neighbors of node v_i is denoted as $N_i=\{v_j:(v_j,v_i)\in\mathcal{E}\}$, *i.e.* the set of nodes with arcs incoming into v_i. Define the in-degree matrix as a diagonal matrix $D=\operatorname{diag}(d_1\ldots d_N)$ with $d_i=\sum_j e_{ij}$ the (weighted) in-degree of node i (*i.e.* the i-th row sum of E). Define the graph Laplacian matrix as $L=D-E$, which has all row sums equal to zero.

A directed path from node v_{i_1} to node v_{i_k} is a sequence of edges $\left(v_{i_1},v_{i_2}\right),\left(v_{i_2},v_{i_3}\right),\ldots\left(v_{i_{k-1}},v_{i_k}\right)$, with $\left(v_{i_{j-1}},v_{i_j}\right)\in\mathcal{E}$ for $j=\{2,\ldots,k\}$. The graph is said to contain a (directed) spanning tree if there exists a vertex such that every other vertex in $\mathcal{V}$ can be connected by a directed path starting from it. Such a special vertex is then called a root.

We denote the real numbers by $\mathbb{R}$, the positive real numbers by $\mathbb{R}^+$, and the complex numbers by $\mathbb{C}$.

2.2. State feedback for synchronization of multi-agent systems

Given a graph $\mathcal{G}(\mathcal{V},\mathcal{E})$, endow each of its N nodes with a state vector $x_i\in\mathbb{R}^n$ and a control input $u_i\in\mathbb{R}^m$ and consider at each node the discrete-time dynamics

$$x_i(k+1)=Ax_i(k)+Bu_i(k). \tag{1}$$

Assume that (A,B) is stabilizable and B has full column rank m.

Consider also a leader node, control node, or command generator

$$x_0(k+1)=Ax_0(k) \tag{2}$$

with $x_0\in\mathbb{R}^n$. For instance, if $n=2$ and A has imaginary poles then the leader trajectory is a sinusoid.

The *cooperative tracker* or *synchronization problem* is to select the control signals u_i, using the relative state of node i to its neighbors, such that all nodes synchronize to the state of the control node, that is, $\lim_{k\to\infty}\|x_i(k)-x_0(k)\|=0,\ \forall i$. These requirements should be fulfilled for all initial conditions $x_i(0)$. If the trajectory $x_0(k)$ approaches a fixed point, this is normally called the consensus problem.

To achieve synchronization, define the local neighborhood tracking errors

$$\varepsilon_i=\sum_{j\in N_i}e_{ij}\left(x_j-x_i\right)+g_i\left(x_0-x_i\right) \tag{3}$$

where pinning gain $g_i\geq0$ is nonzero if node v_i can sense the state of the control node. The intent is that only a small percentage of nodes have $g_i>0$, yet all nodes should synchronize to the trajectory of the control node using local neighbor control protocols (Wang & Chen, 2002). It is assumed that at least one pinning gain is nonzero. Note that the local neighborhood tracking error represents the information available to agent i for control purposes.

Choose the input of agent i as the weighted local control protocol

$$u_i=c\left(1+d_i+g_i\right)^{-1}K\varepsilon_i, \tag{4}$$

where $c\in\mathbb{R}^+$ is a coupling gain to be detailed later. Then, the closed-loop dynamics of the individual agents are given by

$$x_i(k+1)=Ax_i(k)+c\left(1+d_i+g_i\right)^{-1}BK\varepsilon_i(k). \tag{5}$$

Defining global tracking error and state vectors $\varepsilon=[\varepsilon_1^T\ \ldots\ \varepsilon_N^T]^T\in\mathbb{R}^{nN}$, $x=[x_1^T\ \ldots\ x_N^T]^T\in\mathbb{R}^{nN}$, one may write

$$\varepsilon(k)=-(L+G)\otimes I_n x(k)+(L+G)\otimes I_n\bar{x}_0(k) \tag{6}$$

where $G=\operatorname{diag}(g_1,\ldots,g_N)$ is the diagonal matrix of pinning gains and $\bar{x}_0(k)=\underline{1}\otimes x_0(k)$ with $\underline{1}\in\mathbb{R}^N$ the vector of 1's. The global dynamics of the N-agent system is given by

$$x(k+1)=\left[I_N\otimes A-c\left(I+D+G\right)^{-1}(L+G)\otimes BK\right]x(k)$$
$$+c\left(I+D+G\right)^{-1}(L+G)\otimes BK\bar{x}_0(k). \tag{7}$$

Define the *global disagreement error* $\delta(k)=x(k)-\bar{x}_0(k)$ (Olfati-Saber & Murray, 2003). Then one has the global error dynamics

$$\delta(k+1)=A_c\delta(k) \tag{8}$$

where the closed-loop system matrix is

$$A_c=\left[I_N\otimes A-c\left(I+D+G\right)^{-1}(L+G)\otimes BK\right]. \tag{9}$$

We shall refer to matrix

$$\Gamma=\left(I+D+G\right)^{-1}(L+G) \tag{10}$$

as the (weighted) *graph matrix* and to its eigenvalues Λ_k, $k=1,\ldots,N$, as the *graph matrix eigenvalues*. Assume the graph