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Parameter identification of fractional order systems using block pulse functions

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ABSTRACT

In this paper, a novel method is proposed to identify the parameters of fractional-order systems. The proposed method converts the fractional differential equation to an algebraic one through a generalized operational matrix of block pulse functions. And thus, the output of the fractional system to be identified is represented by a matrix equation. The parameter identification of the fractional order system is converted to a multi-dimensional optimization problem, whose goal is to minimize the error between the output of the actual fractional order system and that of the identified system. The proposed method can simultaneously identify the parameters and the fractional differential orders of the fractional order system and avoid the drawbacks in the literature that the fractional differential orders should be known or commensurate. Furthermore, the proposed method avoids complex calculations of the fractional derivative of input and output signals. Illustrative examples covering both fractional and integer systems are given to demonstrate the validity of the proposed method.

1. Introduction

The concept of fractional calculus (FC) was proposed by Leibniz more than 300 years ago [1]. Since it was proposed, FC has been only developed in the field of pure mathematics and almost ignored by scientists and engineers due to its inherent complexity and the fact that it does not have an acceptable geometrical or physical interpretation. With the development of computer science, the calculation of FC has become realizable, and therefore FC has received more and more attention from scientists and engineers in various fields. Compared to integer calculus (IC), FC has two distinct advantages. First, FC is non-local and emphasizes mathematically the long-memory. This characteristic obeys the hereditary character of some physical systems. Many real systems, such as thermal diffusion in a wall [2,3], semi-infinite lossy (RC) transmission line [4], the rotor skin effect of induction machine [5], and viscoelastic systems [6], themselves exhibit fractional behavior. Second, FC provides a concise and accurate way to describe a complex system. It is revealed that many systems can be described more accurately using fractional order models (FOMs). The motion of human beings [7,8], biological tissues [9], education evaluation [10], thermal process [11], the electrical characteristics of solid oxide fuel cells (SOFC) [12], the elastoplastic behavior of metals [13], the giving up smoking behavior [14], the behavior of inhomogeneous anisotropic viscoelastic bodies [15], and neutron transport in a nuclear reactor [16] are all well described by FOM.

In the communication of control, building an effective and accurate model for a system is an important issue.

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Promoted by the advantages of FOM, researchers explore the building of FOM for control systems. For example, in [17], Podlubny modeled heating furnace using IOM and FOM, and comparison results show that the response of FOM is more exact than that of IOM. In [18], a lead acid battery was modeled by a FOM and a parameter estimation method was presented. Wang et al. built a FOM for a thermal process in a boiler main steam system [11]. It is revealed that a FOM is more effective and accurate than the integer order model (IOM). Recently, building FOM for control systems has received more and more attention from researchers. However, because the geometric and physical interpretation of FC is not as distinct as integer calculus (IC), it is difficult to model real systems using FC based on mechanism analysis. Therefore, system identification becomes a practical way for modeling a system using FC.

Fractional order system identification is a process of establishing a FOM capable of a reproducing system's physical behavior as faithfully as possible from a series of observations [19]. Fractional system identification was initially studied by Lay [20], Lin [21], Cois [22] and Aoun [23]. In their studies, two classes of basic method, i.e., equation-error-based method and output-error-based method, had been proposed. In the equation-error-based method, the fractional differential orders are assumed to be fixed or to be commensurate. In this method, the parameters are estimated through minimizing the error between the output of the actual system and that of the estimated system. It involves the computation of fractional derivatives of input/output signals from sampled data by applying the Grünwald–Letnikov (G–L) definition of fractional derivatives. To eliminate the effect of noise, a state vector filter was used to filter the input and output data. The output-error-based method allows simultaneously estimating the fractional differential orders and model's parameters. In the output-error-based method, the error criterion function is constructed according to the output of the system, and the fractional differential orders are assumed to be commensurate. The parameters are iteratively estimated using gradient-based optimization algorithms. A survey of early works about fractional system identification is given in [24,25]. Recently, the simplified refined instrumental variable for continuous-time systems (SRIVC) method was extended to identify FOM in [26]. In [27], a subspace method was proposed to identify a continuous-time fractional system, which is represented by a state space model. In the proposed method, the parameter matrices were identified using the conventional subspace-based technique and the commensurate orders were estimated using nonlinear programming. In [28], Liao et al. also investigated the subspace identification of commensurate fractional systems, in which the estimation bias was eliminated by the instrumental variable (IV) method. In [29], Malti proposed to use the set membership method to estimate the parameters of a fractional model in frequency domain. In [30], a nonlinear fractional order system identification was performed based on Takagi–Sugeno fuzzy model, in which a fractional local model was applied. In [31], a hybrid fractional Box–Jenkins model was identified using the instrumental variable method. In [32], the identification of the continuous-time fractional order model with time delay was investigated, where the instrumental variable was used. As well as the above methods, fractional orthogonal basis functions such as fractional Laguerre basis, Kautz basis were also proposed to identify fractional order systems [33]. In [34], fractional orthogonal rational functions were used to represent fractional system, and a method for estimating the order was presented. In [35], the orthogonal basis functions were extended to fractional differentiation equation by applying the Gram Schmidt process. In addition, identifying fractional system in frequency domain was also studied in [36–40].

However, the methods mentioned above have several disadvantages. First, the fractional differential orders must be known or commensurate. Second, estimating the parameters of a fractional model involves the calculation of the fractional derivative of input and output signals, which is an extensive computation burden. In this paper, we propose a novel method for parameter identification of a fractional order system. The proposed method is based on the operational matrix representation of a fractional differential operator. The operational matrix of the fractional differential operator is obtained through an orthogonal basis functions, i.e., block pulse functions. The operational matrix representation of the fractional differentiation operator converts a differential operation to an algebraic operation, and thus it can not only reduce the computation complexity, but also, probably the most important, make it possible to identify the fractional system with arbitrary orders.

The remainder of the paper is organized as follows. In Section 2, a brief mathematical background of fractional calculus is presented and piecewise orthogonal functions and their generalized operational matrices are also presented in this section. In Section 3, the proposed parameter identification method based on operational matrix is presented in detail. In Section 4, numerical examples are given. The concluding remarks are drawn in Section 5.

2. Mathematical background

2.1. Definitions of fractional derivatives and integrals

Fractional calculus is a generalization of the integration and differentiation to a non-integer order fundamental operator ${}_aD_t^\alpha$, where a and t are the limits and α ($\alpha \in R$) is the order of the operation [41]. The operator is defined as

$$ {}_aD_t^\alpha = \begin{cases} \dfrac{d^\alpha}{dt^\alpha}, & \alpha > 0 \\ 1, & \alpha = 0 \\ \int_a^t (d\tau)^{-\alpha}, & \alpha < 0. \end{cases} \tag{1} $$

There are several definitions for fractional calculus. Among these definitions, the G–L definition and the Riemann–Liouville (R–L) definition are commonly used. The G–L definition is given as

$$ {}_aD_t^\alpha f(t) = \lim_{h \to 0} \frac{1}{h^\alpha} \sum_{j=0}^{[t-a]/h} \binom{q}{j} f(t-jh), \tag{2} $$

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