Automatica 49 (2013) 441–447

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Some results on the stabilization of switched systems ☆

José Luis Mancilla-Aguilar [1], Rafael Antonio García

Department of Mathematics, Instituto Tecnológico de Buenos Aires, Argentina

ARTICLE INFO

Article history:
Received 12 August 2011
Received in revised form
21 March 2012
Accepted 27 August 2012
Available online 1 December 2012

Keywords:
Switched systems
Stabilization
Stability
Asymptotic controllability

ABSTRACT

This paper deals with the stabilization of switched systems with respect to (w.r.t.) compact sets. We show that the switched system is stabilizable w.r.t. a compact set by means of a family of switched signals if and only if a certain control affine system whose admissible controls take values in a polytope is asymptotically controllable to that set. In addition we present a control algorithm that based on a family of open-loop controls which stabilizes the aforementioned control system, a model of the system and the states of the switched system, generates switching signals which stabilize the switched system in a practical sense. We also give results about the convergence and the robustness of the algorithm.

1. Introduction

Switched systems are a special class of hybrid systems and have numerous applications in many fields (see Liberzon, 2003; Liberzon & Morse, 1999; Matveev & Savkin, 2000; van der Schaft & Schumacher, 2000). Mathematically, a switched system can be described by a differential equation of the form

$$\dot{x}(t) = f_{\sigma(t)}(x(t)), \tag{1}$$

where $\mathcal{F} = \{f_i : \mathbb{R}^n \to \mathbb{R}^n : i = 1, \ldots, N\}$ is a finite family of sufficiently regular vector fields and where $\sigma : [0,\infty) \to \{1, \ldots, N\}$ is the switching signal, i.e. σ is a piecewise constant and continuous from the right function.

In Liberzon and Morse (1999), Lin and Antsaklis (2007) and Shorten, Wirth, Mason, Wulff, and King (2007), some basic problems related to stability issues are surveyed, among which we note, in particular, the so-called stabilization problem, which we roughly state as follows (Problem C in Liberzon & Morse, 1999): *Construct switching signals that make the origin an asymptotically stable point of the switched system.*

A popular approach to solve this problem, which we call the *closed-loop approach*, basically consists in finding a state-dependent switching rule $k : \mathbb{R}^n \to \{1, \ldots, N\}$ such that with $\sigma(t) = k(x(t))$, the closed-loop system

$$\dot{x}(t) = f_{k(x(t))}(x(t)) = g(x(t)) \tag{2}$$

is globally asymptotically stable at $x = 0$. Since any such a map k is necessarily discontinuous two problems arise: (i) the closed-loop system (2) may not have classical solutions for some initial conditions (a classical or Caratheodory solution of (2) is a locally absolutely continuous function $x : [0, T) \to \mathbb{R}^n$, such that $\dot{x}(t) = g(x(t))$ for almost all $t \in [0, T)$); (ii) for some classical solutions $x(t)$ of (2), $\sigma(t) = k(x(t))$ may not necessarily be a switching signal since, for example, σ could have a point of accumulation of switchings times (Zeno behavior) or even a more complicated set of discontinuities (see Ceragioli, 2006). Of course one can consider generalized solutions of (2) (for instance Filippov or Krasovskii ones) to overcome (i), but some of these generalized solutions $x(t)$ of (2) might not be a solution of (1) for any switching signal $\sigma(t)$ since they exhibit 'chattering'.

The switching rule k is usually constructed with the help of a Lyapunov function V (also called weak or control Lyapunov function) or a family of them (see Bacciotti, 2004; Liberzon, 2003; Lin & Antsaklis, 2007; Liu, Liu, & Xie, 2010 and the references therein) and it is implemented by using some kind of hysteresis in order to avoid both Zeno behavior and chattering. In this regard, it is pertinent to note that the discontinuous feedback stabilizers constructed for general nonlinear systems in Clarke, Ledyaev, Rifford, and Stern (2000); Clarke, Ledyaev, Sontag, and Subbotin (1997) and Kellet and Teel (2004) by using a control Lyapunov function of the system (which always exists if the system is asymptotically controllable, Clarke et al., 1997) semi-globally stabilize the switched system in a practical sense when they are implemented by means of sampling and zero-order hold. One of

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Michael Malisoff under the direction of Editor Andrew R. Teel.

E-mail addresses: jmancill@itba.edu.ar (J.L. Mancilla-Aguilar), ragarcia@itba.edu.ar (R.A. García).

[1] Tel.: +54 11 63934800; fax: +54 11 63934811.

0005-1098/$ – see front matter
doi:10.1016/j.automatica.2012.11.002

the main drawbacks of the closed-loop approach is that one usually needs to find suitable Lyapunov functions for designing the state-dependent stabilizing switching rule. Besides the fact that it is not easy to find such functions, they may not belong to a "nice" class of functions. For example, it was recently proven in Blanchini and Savorgnan (2008) that some simple stabilizable planar switched linear systems do not admit a convex Lyapunov function.

Motivated by the discussion above, this work considers an alternative approach, which we call the *open-loop approach*, to solve the stabilization problem. It basically consists in finding a parameterized family of switching signals $\Sigma = \{\sigma_{x_0}\}_{x_0\in\mathbb{R}^n}$, such that σ_{x_0} asymptotically drives the initial state x_0 to the origin in a suitable manner. This approach was less explored than the closed-loop one, and only a few works followed it. Some results were reported in Sun and Ge (2005) (see also the references therein) and in Bacciotti and Mazzi (2012) for switched linear systems, and in Bacciotti and Mazzi (2010) for switched nonlinear systems. One of the main drawbacks of this approach is the lack of robustness of the solutions so obtained, mainly due to measurement errors in the initial conditions and modeling errors in the system dynamics. On the other hand, it does not exhibit the well-posedness problems mentioned for the closed-loop one and it is not necessary the knowledge of Lyapunov functions for designing the stabilizer Σ.

In this paper we explore the open-loop approach for a more general problem: the stabilization of a switched system w.r.t. a compact set, (see Goebel, Sanfelice, & Teel, 2009 for a motivation to stabilization w.r.t. compact sets rather than a point). To this end, in Section 2 we embed the switched system into a control affine nonlinear one with controls taking values in a convex polytope, and show that the problem can be solved for the switched system if and only if it can be solved for the control system, which is a better studied problem (see for instance Colonius & Kliemann, 2000, chapter 12) and, *a priori*, easier to solve due to the structure of the control set. In Section 3 we present an algorithm that, based on an open- or closed-loop solution of the stabilization problem for the control system, generates switching signals that stabilize the switching system in a practical sense. An interesting feature of this algorithm is that it is robust with respect to small errors in the measurements of the states and small uncertainties in the vector fields f_i. So, the implementation of an open-loop solution Σ by this method precludes to a certain degree the drawback mentioned above. The results in Sections 2 and 3 suggest an alternative approach to the design of switching laws for stabilizing a switching system w.r.t a compact set, that consists in (a) to design a stabilizer for the control system (by using the various well-established design techniques) and (b) to obtain the stabilizing switching signal via the proposed algorithm. In Section 4 we illustrate the obtained results by means of an example and finally, Section 5 contains some conclusions.

2. Open-loop stabilizability

In what follows we suppose that the vector fields of the family $\mathcal{F}$ which gives rise to the switched system (1) are locally Lipschitz and that $\mathcal{A}$ is a nonempty compact subset of $\mathbb{R}^n$. For a subset $B \subset \mathbb{R}^n$, we denote by $|x|_B$ the distance from $x \in \mathbb{R}^n$ to B, i.e. $|x|_B = \inf_{b\in B} |x - b|$, where $|\cdot|$ is the Euclidean norm on $\mathbb{R}^n$.

In order to study the stabilizability of (1) w.r.t. $\mathcal{A} \subset \mathbb{R}^n$, we embed the switched system into the control system

$$\dot{z}(t) = \sum_{i=1}^{N} u_i(t)f_i(z(t)) := F(z(t))u(t) \tag{3}$$

where $F(z) = [f_1(z) \cdots f_N(z)] \in \mathbb{R}^{n\times N}$ and for $t \geq 0, z(t) \in \mathbb{R}^n$ and $u(t) \in U = \text{co}(U^*)$, with $U^* = \{e_1, \ldots, e_N\}$. Here $e_i \in \mathbb{R}^N$ denotes the i-th canonical vector of $\mathbb{R}^N$ and co(B) is the convex hull of a subset $B \subset \mathbb{R}^N$.

We assume that the admissible controls of (3) belong to $\mathcal{U}$, the set of all the Lebesgue measurable functions $u : [0, \infty) \to U$, and denote by $\mathcal{U}^*_{pc}$ the subclass of all $u \in \mathcal{U}$ that take values in U^* and are piecewise constant and continuous from the right. For $z_0 \in \mathbb{R}^N$ and $u \in \mathcal{U}$, $z(\cdot, z_0, u)$ will denote the unique maximally defined solution of (3) which verifies $z(0, z_0, u) = z_0$.

By considering the bijective correspondence between the set of switching signals of (1) and $\mathcal{U}^*_{pc}$, $\sigma \mapsto u_\sigma$ with $u_\sigma(t) = e_{\sigma(t)}$ for all $t \geq 0$, and taking into account that for each $x_0 \in \mathbb{R}^n$ and each switching signal σ, $x(\cdot) = z(\cdot, x_0, u_\sigma)$ is the unique maximally defined solution of (1) corresponding to σ which verifies $x(0) = x_0$, we can identify the switched system (1) with the control system (3) with admissible controls restricted to $\mathcal{U}^*_{pc}$. This embedding was used, for example, in Bengea and DeCarlo (2005) to solve optimal control problems for switched systems.

In order to study the stabilization of (3) w.r.t. $\mathcal{A}$ by means of controls in $\mathcal{U}^*_{pc}$, we introduce the following.

Definition 1. Let $\mathcal{U}'$ be a subclass of $\mathcal{U}$. The control system (3) is $\mathcal{U}'$-stabilizable w.r.t. $\mathcal{A}$ if there exists a parameterized family $\Sigma = \{u_{z_0}\}_{z_0\in\mathbb{R}^N\setminus\mathcal{A}}$ of controls in $\mathcal{U}'$ such that for some function $\beta \in \mathcal{KL}$[2]

$$|z(t, z_0, u_{z_0})|_\mathcal{A} \leq \beta(|z_0|_\mathcal{A}, t) \quad \forall t \geq 0, \forall z_0 \in \mathbb{R}^n \setminus \mathcal{A}. \tag{4}$$

In addition, Σ will be referred to as a $\mathcal{U}'$-stabilizer of (3).

Remark 2. At first glance, a seemingly more natural definition of $\mathcal{U}'$-stabilizability is obtained by asking for the existence of a family of parameterized controls $\Sigma = \{u_{z_0}\}_{z_0\in\mathbb{R}^n}$ which verifies (4) for all $z_0 \in \mathbb{R}^n$. Nevertheless, unless $\mathcal{U}' = \mathcal{U}$ (see Proposition 5), such a definition is too restrictive. For example, the control system (3), with $n = 1, N = 2, f_1(z) = -1, f_2(z) = 1$ and $\mathcal{A} = \{0\}$, is $\mathcal{U}^*_{pc}$-stabilizable in the sense of Definition 1 but it is not if we adopt one which requires that (4) holds also when $z_0 = 0$, since there is no control $u \in \mathcal{U}^*_{pc}$ such that $|z(t, 0, u)| = \beta(0, t) = 0$ for all $t \geq 0$. In connection with this, we note that Definition 1 allows us to face the stabilization problem of a switched system w.r.t. a point x_e without the (usual) assumption that $f_i(x_e) = 0$ for some i.

As for the fact that the controls are not defined for initial conditions in $\mathcal{A}$, in a practical situation the $\mathcal{U}'$-stabilizer could be applied as follows when $z_0 \in \mathcal{A}$: fix $\varepsilon > 0$. Pick any control $v \in \mathcal{U}'$ and apply it while $|z(t, z_0, v)|_\mathcal{A} < \varepsilon$. If t^* is the first time such that $z(t^*, z_0, v) = z_1$ verifies $|z_1|_\mathcal{A} = \varepsilon$, then apply the control $u(t) = u_{z_1}(t-t^*)$ on the interval $[t^*, \infty)$. In this way we can obtain the stabilization of (3) w.r.t an arbitrary neighborhood of $\mathcal{A}$.

Remark 3. If $k : \mathbb{R}^n \to U$ is a continuous map such that the closed-loop system

$$\dot{z} = F(z)k(z) \tag{5}$$

is globally uniformly asymptotically stable w.r.t. $\mathcal{A}$, the parameterized family of controls $\Sigma = \{u_{z_0}\}_{z_0\in\mathbb{R}^n\setminus\mathcal{A}}$, with $u_{z_0}(\cdot) := k(z(\cdot))$, where $z(\cdot)$ is a solution of (5) which satisfies $z(0) = z_0$, is a $\mathcal{U}$-stabilizer of (3).

Remark 4. With a proof similar to that of Proposition 2.3 in Albertini and Sontag (1999), it follows that $\Sigma = \{u_{z_0}\}_{z_0\in\mathbb{R}^N\setminus\mathcal{A}}$ is an $\mathcal{U}'$-stabilizer of (3) if and only if the following hold:

(1) For all $z_0 \in \mathbb{R}^n \setminus \mathcal{A}$, $z(\cdot, z_0, u_{z_0})$ is defined for all $t \geq 0$;
(2) (Lyapunov stability) there exists a function M of class $\mathcal{K}_\infty$ such that for every $R > 0$ and every $z_0 \in \mathbb{R}^n \setminus \mathcal{A}$ with $|z_0|_\mathcal{A} \leq R$,

$$|z(t, z_0, u_{z_0})|_\mathcal{A} \leq M(R) \quad \forall t \geq 0.$$

[2] As usual, by a $\mathcal{K}_\infty$-function we mean a continuous function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ that is strictly increasing and unbounded, and satisfies $\alpha(0) = 0$ and $\mathcal{KL}$ is the set of functions $\mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ that are of class $\mathcal{K}_\infty$ in the first argument and decrease to 0 when the second argument goes to ∞.