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# Fractional model of an electrochemical capacitor

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ABSTRACT

The fractional model of the electrochemical capacitor (EC) and its potential relaxation are presented. The potential relaxation occurs after charge or discharge current interruption. The EC fractional model is based on the fractional order transfer function that was obtained by means of least squares fitting of the EC impedance data. The inverse Laplace transform is used to obtain the EC impulse response. By using of the EC impulse response the EC charge and discharge simulation were performed.



## 1. Introduction

### 1.1. On the fractional derivatives

The Fractional Calculus (FC) is a generalization of the traditional calculus that leads to similar concepts and tools, but with wider generality and applicability [1–6]. By allowing derivative and integral operations of arbitrary real or complex order, it enlarges the applicability and modeling ability.

Here, we will make a brief introduction to the systems described by constant coefficient linear fractional differential equations: fractional linear time-invariant (FLTI) systems [1–3]. They assume the general format as follows:

$$\int_{n=0}^{N} a_n D^{\alpha_n} y(t) = \int_{m=0}^{M} b_m D^{\alpha_m} x(t) \text{ when } \alpha_n < \alpha_{n+1}, \tag{1}$$

where D means derivative and $\alpha_n$ $(n=0, 1, 2, \ldots)$ are derivative orders that we will assume to be positive real numbers.

With this definition, we are in conditions to define and compute the Impulse Response and Transfer Function. As any other shift-invariant linear system the system described by (1) has the exponential as eigenfunction. Letting $x(t)=e^{st}$, where $s \in C$ and $t \in R$, we obtain $y(t)=H(s)e^{st}$, where $H(s)$ is the transfer function given by

$$H(s) = s^{\alpha} \tag{2}$$

provided that $\mathrm{Re}(s) > 0$ or $\mathrm{Re}(s) < 0$.

With $s=j\omega$, we obtain the Frequency Response, $H(j\omega)$, and we can represent the Bode diagrams as in the usual systems. The asymptotic amplitude Bode diagrams are constituted by straight lines with slopes that may assume any value, contrarily to the usual case where the slopes are multiples of 20 dB/decade, as it can be seen below in our study.

The system represented by $s^{\alpha}$ is called differintegrator [1–3]. However, we must be careful when dealing with $s^{\alpha}$ that is a multivalued expression defining an infinite number of Riemann surfaces. Each Riemann surface defines one function. Therefore, (1) can represent an infinite number of linear systems. However, only the principal Riemann surface may lead to a real system. Constraining $s^{\alpha}$ by imposing a region of

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convergence, we define a transfer function. Choosing the left half real axis as branch cut line we obtain the transfer function of the causal system. Its impulse response is given by

$$\delta_{+}^{(\alpha)}(t) = \frac{t^{-\alpha-1}u(t)}{\Gamma(-\alpha)}, \quad (3)$$

where $\Gamma(.)$ is the Euler gamma function.

With $\alpha = -1$, we obtain the normal integrator impulse responses. With those impulse responses, we can obtain fractional the differintegrated of a given signal by the convolution. We are led to the forward:

$$D_{+}^{(\alpha)}f(t) = \frac{1}{\Gamma(-\alpha)}\int_{-\infty}^{t} f(\tau)(t-\tau)^{-\alpha-1}d\tau. \quad (4)$$

Although we can use this relation for defining fractional derivative this is not convenient from analytical point of view. The simplest way of doing it is from the generalized difference. It is called forward Grünwald–Letnikov derivative and is given by [1–3]:

$$D_{d}^{\alpha}f(z) = \lim_{h\to 0+} \frac{\sum_{k=0}^{\infty}(-1)^{k}\binom{\alpha}{k}f(z-kh)}{h^{\alpha}}, \quad (5)$$

where $h$ is any complex number in the right hand complex plane.

To exemplify let us apply it to the function $f(z) = e^{az}$. If $\text{Re}(a) > 0$, expression (5) converges to $D^{\alpha;d}f(z) = a^{\alpha} e^{az}$. It is interesting to remark that, if $z$ and $h$ are real, in (5) we are using the current and past values of the function: it is a causal derivative. There is also a backward (non-causal derivative) [1–3].

### 1.2. The Electrochemical Capacitors

The Electrochemical Capacitors (ECs), also known as double layer capacitors, supercapacitors or ultracapacitors, accumulate the electric energy like the traditional capacitors. However, in contrast with the conventional capacitors, the ECs:

(a) have a large capacitance (of the order of thousand Farad);
(b) have very small active resistance (some mΩ or hundreds μΩ);
(c) are able to perform several hundred thousand charge and discharge cycles;
(d) have the outstanding characteristic of high power delivery;
(e) have long useful life;
(f) its power density is considerably higher than that of batteries.

The electrical characteristics of ECs makes them suitable for a variety of current applications such as hybrid electric vehicle (EV), power electronics and telecommunications, where short, high-power pulses are required. The ECs may also operate in combination with batteries, solar and fuel cells [7].

The main difference between ECs and conventional capacitors lies in the energy storage. The ECs use the Electric Double Layer (EDL) for accumulating of the electric energy. Each consists of two electrodes and the space between them is filled with an electrolyte. At the middle there is a separator as shown in Fig. 1.

The main disadvantage of ECs is the low operational voltages from 2.7 V to 3.3 V. To overcome this disadvantage the ECs are connected in serial and packed in modules. In this case it is very important to sort out ECs with similar electrical parameters unless they are charged with different speeds. For example EC number 2 that is shown in Fig. 2 has lowest capacitance and highest active resistance.

Other ECs are not charged completely and the total module voltage is less than the working module voltage. In this case we continue to charge the module and the voltage of the EC number 2 exceed the critical value. That leads to the damage of EC number 2 and then to the damage of the module.

To prevent exceeding the critical values of voltage for all EC in the module it is necessary to measure the

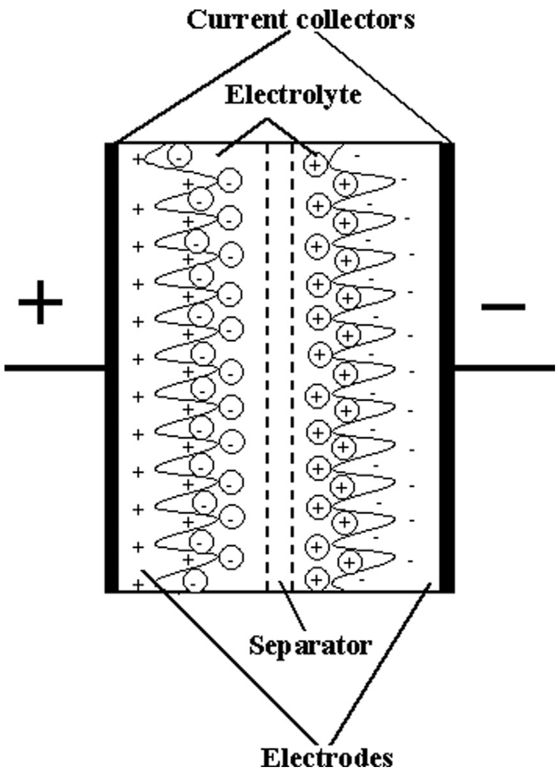


**Fig. 1.** Construction of ECs.

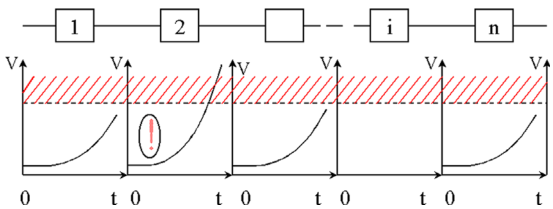


**Fig. 2.** Exceeding critical values of voltage for EC number 2.