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Analysis of radial composite systems based on fractal theory and fractional calculus

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ABSTRACT

This paper analyzes the mathematical modeling of a two-region composite reservoir using the concepts of fractal geometry and fractional calculus. Heterogeneity of the reservoir is considered based on fractal geometry. Fractional calculus is used to consider production history in fractal reservoirs. An analytical solution is derived for the pressure-transient behavior of a well in a radial composite system when wellbore storage and skin effects are significant. Some new type-curves are developed under three outer boundary conditions: infinite, closed and constant-pressure. These new type curves can be used to analyze well-tests from a variety of enhanced oil recovery projects, geothermal reservoirs and acidization projects, more accurately.

1. Introduction

The majority of the conventional pressure-transient models assume that permeability and porosity of a formation are invariant in space. These models can be used in an appropriate approach only if the variations of permeability and porosity are small compared with the wellbore permeability and wellbore porosity. However, core, well logging, outcrop data, production behavior, and the dynamic behavior of these reservoirs indicate that many types of reservoirs cannot be justified by these assumptions. Fractal geometry is a suitable tool to describe the response of a fractal reservoir; especially it is appropriate to scale the extreme non-uniformity and consequence of porous media. Starting with the seminal work of O'Shaughnessy and Procaccia [19] and Chang and Yortsos [20], fractal theory has been successfully applied to pressure-transient testing. However, in some sense, the fractal geometry fails to consider the non-locality of the media and so cannot explain the complex dynamics processes on such kind of media [1–7].

On the other hand, fractional calculus, and so called fractional operators, is known since the early 17th century. The more interesting property of such operators, which extend the classical integral and derivative operators, for their applications is that they are non-local involving a power type memory [7–9]. Fractional operators have been extensively applied in many applied fields which have seen an overwhelming growth in the last three decades. A few examples are in physics, engineering, bioengineering, or communication systems [8–18]. Actually, in many cases, it is clear that the fractional models are more suitable for engineering systems than the counterpart based on ordinary derivative approach. In reservoir engineering, Metzler et al. [21] use the concept of fractional derivative to capture the history and non-locality of flow as memory in fractal-fractional diffusion (FFD) model [22–24].

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This paper presents methods for analysis of pressure-transient behavior of radial composite systems based on fractal geometry and fractional derivative concepts. Some new type curves are generated with this technique to analyze well test data from radial composite reservoirs and to determine the well and reservoir parameters more accurately.

The organization of this paper is as follows: Section 2 describes the basic concepts of fractional calculus. The model description is provided in Section 3. In Section 4, the wellbore pressure is computed in Laplace space. Section 5 describes the new type curves generated by the analytical solutions. Finally, the conclusions are given in Section 6.

2. Fractional calculus

In this section some preliminaries, that support the following sections, are presented, (, Refs. [8,15,25,26]). As a generalization of the Cauchy n-tuple integral formula, fractional order integral operator of a continuous function $f(t)$ can be defined as follows:

$$I_{a+}^{q}x(t)=\frac{1}{\Gamma(q)}\int_a^t (t-s)^{q-1}f(s)ds,(q\in R^+,a\in R) \tag{1}$$

in which $\Gamma(q)=\int_0^\infty e^{-z}z^{q-1}dz, q>0$ is the Gamma function.

The corresponding left fractional derivative of a continuous function $f(t)$ in the sense of Riemann–Liouville (RL) is defined as follows:

$$^{RL}D_{a+}^{q}f(t)=D^mI_{a+}^{m-q}f(t) =\frac{1}{\Gamma(m-q)}\frac{d^m}{dt^m}\int_a^t (t-s)^{m-q-1}f(s)ds,(q\in \mathbf{R}^+,a\in \mathbf{R}) \tag{2}$$

where $\lceil q\rceil=m\in\mathbf{Z}^+$.

The left fractional order derivative operator in the sense of Caputo is defined as follows:

$$^{C}D_{0+}^{q}f(t):=I_{0+}^{m-q}D^mf(t)=\frac{1}{\Gamma(m-q)}\int_a^t (t-s)^{m-q-1}f^{(m)}(s)ds, \tag{3}$$

where $q\in\mathbf{R}^+$ and $\lceil q\rceil=m\in\mathbf{Z}^+$.

As a comparison that shows the preference of Caputo definition in engineering applications, the Laplace transforms of these two definitions are stated as follows:

1. $L\{^{C}D_{0+}^{q}f(t)\}=s^qF(s)-\sum_{k=0}^{m-1}s^{q-k-1}f^{(k)}(0)$
2. $L\{^{RL}D_{0+}^{q}f(t)\}=s^qF(s)-\sum_{k=0}^{m-1}s^{k\,RL}D_{0+}^{q-k-1}f(0)$,

in which $m-1<q<m\in\mathbf{Z}^+$ and $F(s)$ is the Laplace transform of $f(t)$. From the first property it can be seen that for evaluating the Laplace transform of a Caputo differentiated function, the integer order derivatives of the function at the initial time are needed whereas for the Riemann–Liouville ones, the fractional order derivatives of the function are needed for computing the Laplace transform. Thus, for the problem considered in this work the Caputo definition looks like the suitable fractional derivative to be used, although other suitable alternative could be used like the fractional derivative called Grünwald–Letnikov.

3. Model description

The FFD model for a fractally radial composite system involving the fractional Caputo derivative could be written, in accordance with [21,27,28], as the following two fractional differential equations (dual fractional differential equation):

$$\frac{1}{r_D^\theta}\frac{\partial^2 p_{D1}}{\partial r_D^2}+\frac{\beta}{r_D^{\theta+1}}\frac{\partial p_{D1}}{\partial r_D}=\frac{\partial^\gamma p_{D1}}{\partial t_D^\gamma},\quad \text{for } 1\le r_D\le R_D, \tag{4}$$

and

$$\frac{1}{r_D^\theta}\frac{\partial^2 p_{D2}}{\partial r_D^2}+\frac{\beta}{r_D^{\theta+1}}\frac{\partial p_{D2}}{\partial r_D}=\frac{\partial^\gamma p_{D2}}{\partial t_D^\gamma},\quad \text{for } R_D\le r_D\le r_{eD}<\infty \tag{5}$$

where $\beta=d_{mf}-\theta-1$, $\gamma=2/(2+\theta)$ and $\partial^\gamma p_D/\partial t_D^\gamma$ are given by Eq. (3). Here d_{mf} and θ are the mass fractal dimension and conductivity index, respectively. Since $0\le\theta$, so $0<\gamma\le 1$. Initial conditions are $p_{D1}(r_D,0)=0$, and $p_{D2}(r_D,0)=0$.

Inner boundary condition without wellbore storage and skin effects is $(r_D^\beta(\partial p_{D1}/\partial r_D))_{r_D=1}=-1$.

Conditions at the discontinuity are

$$\frac{\partial p_{D2}}{\partial r_D}=M\frac{\partial p_{D1}}{\partial r_D},\quad \text{for } r_D=R_D;t_D>0 \text{ and } p_{D2}=p_{D1},$$
for $r_D=R_D; t_D>0$.

Outer boundary conditions are as follows:Infinite: $\lim_{r_D\to\infty}p_D(r_D,t_D)=0$.Closed: $(\partial p_D/\partial r_D)_{r_D=r_{eD}}=0$.Constant pressure: $p_D(r_{eD},t_D)=0$,where the dimensionless variables are as defined below.Dimensionless pressure:

$$p_{D1}=\frac{k_1h}{141.2qB\mu_1}(p_i-p_1(r,t)),$$
$$p_{D2}=\frac{k_2h}{141.2qB\mu_2}(p_i-p_2(r,t)),$$
$$p_{wD}=\frac{k_1h}{141.2qB\mu_1}(p_i-p_w(t))$$

where k, h, q, B, and μ represent the reservoir rock permeability, net formation thickness, flow rate, liquid formation volume factor and liquid viscosity, respectively. And p_i , p and p_w denote the initial reservoir pressure, reservoir pressure and wellbore pressure, respectively.Mobility ratio is $M=((k/\mu)_1/(k/\mu)_2)$, Hydraulic diffusivity ratio is $D=(k/\phi\mu c_t)_1/(k/\phi\mu c_t)_2$, where ϕ denotes the reservoir rock porosity.Dimensionless time is $t_D=0.0002637\,k_1t/(\phi\mu c_t)_1r_w^2$, where t is the elapsed time and c_t and r_w represent the total compressibility and wellbore radius, respectively. Dimensionless radii are $r_D=(r/r_w)$, $R_D=(R/r_w)$, and $r_{eD}=(r_e/r_w)$, where r, R, and r_e denote the radial distance, radius of discontinuity, and external radius, respectively.

Dimensionless wellbore storage coefficient: $C_D=5.6146\,C/2\pi(\phi c_t)_1hr_w^2$, where C is the wellbore storage coefficient.

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