Automatica 49 (2013) 519–525

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Rendezvous in space with minimal sensing and coarse actuation[☆]

Soumya Ranjan Sahoo, Ravi N. Banavar [1], Arpita Sinha

Systems and Control Engineering, IIT Bombay, Mumbai, India

ARTICLE INFO

Article history:
Received 7 September 2011
Received in revised form
29 June 2012
Accepted 9 September 2012
Available online 8 December 2012

Keywords:
Consensus
Rendezvous
Minimal sensing
Quantized control
UAVs

ABSTRACT

In this paper, we propose a control law to achieve a rendezvous of autonomous vehicles moving in three-dimensional (3D) space, using minimal data sensing and quantized control. A pre-assigned graph uniquely assigns the pursuer-target pair in a cyclic manner. A quantized control law has been proposed which allows the vehicle to pitch and yaw simultaneously in the required direction and track its target agent. The *only* measurement required for the proposed control law is the quadrant from which the target vehicle moves out of the field-of-view of the pursuing vehicle. A Lyapunov function is chosen to find a domain for the field-of-view which guarantees rendezvous under the proposed control law. Computer simulations are presented to demonstrate the control law.

1. Introduction

Autonomous vehicle systems have found potential applications in various military and civil operations. Greater benefits arise from cooperation of a team of vehicles than individual. A multi-agent system is robust to failure and more efficient than individual agents in certain cases. It is also possible to reduce the size and operational cost of individual agents and increase system reliability. This has aroused interest in the control community in cooperative control and consensus algorithms. In Ren, Beard, and Atkins (2005), the authors have mentioned various consensus algorithms in multi-agent coordination.

The rendezvous problem is one of the various consensus problems where all agents of a multi-agent system converge to a point at the same time. Rendezvous of autonomous agents using various decentralized or distributed controls has been pursued actively in the past few decades. Some papers which discuss controllers for rendezvous are Ando, Oasa, Suzuki, and Yamashita (1999), Hui (2011) and Lin, Morse, and Anderson (2007a,b). The cyclic pursuit problem is closely related to the rendezvous problem. There is a vast literature on control for cyclic pursuit, some of which are Bruckstein, Cohen, and Efrat (1991), Galloway, Justh, and Krishnaprasad (2009, 2010), Marshall, Broucke, and Francis (2004a,b), Pavone and Frazzoli (2007); Ramirez, Pavone, and Frazzoli (2009), Richardson (2001) and Sinha and Ghose (2006). The pursuit curves have been studied in Bernhart (1959). In the references cited above, information regarding relative position, angle or velocity is required to achieve the objective. Sometimes tasks have to be done with minimal data and quantized controllers. We mention some of the papers which look into minimalism and quantized control applications in the following parts.

Minimalism means, given an objective to be achieved by a group of autonomous agents, what is the minimum information needed to achieve the objective? Optimal navigation, pursuit–evasion, robot localization with minimal data have been discussed in Fredslund and Matarić (2002), Sachs, LaValle, and Rajko (2004); Tovar, Guilamo, and LaValle (2005); Tovar, LaValle, and Murrieta (2003); Tovar, Murrieta-Cid, and LaValle (2007). Various sensorless manipulation tasks have been explored in Böhringer, Brown, Donald, Jennings, and Rus (1997). The use of quantized control and coarse quantized measurement of plant outputs (states) is motivated by restricted information flow between plant and controller for various reasons. While the use of quantized measurements for stabilization has been discussed in Brockett and Liberzon (2000) and Delchamps (1990); Ishii and Francis (2002), Nair and Evans (2000) and Petersen and Savkin (2001) discuss the controllers with a finite data rate communication link. In Elia and Mitter (2001), the authors have presented the coarsest, least dense quantizers for state-feedback controller and estimator to stabilize a single-input–single-output linear time-invariant system.

The above cited references motivated us to look into the application of minimalism and quantized control to a multi-agent

[☆] The material in this paper was partially presented at the 50th IEEE Conference on Decision and Control (CDC 2011) and European Control Conference (ECC), Dec. 12–15, 2011, Orlando, Florida, USA. This paper was recommended for publication in revised form by Associate Editor Jun-ichi Imura under the direction of Editor Toshiharu Sugie.

E-mail addresses: soumya@sc.iitb.ac.in (S.R. Sahoo), banavar@sc.iitb.ac.in (R.N. Banavar), asinha@sc.iitb.ac.in (A. Sinha).

[1] Tel.: +91 22 2576 7888; fax: +91 22 2572 0057.

0005-1098/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2012.11.024

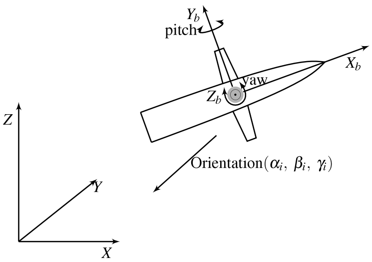

Fig. 1. Schematic of vehicle and transformation from body-fixed frame to earth-fixed frame.

system. In Yu, LaValle, and Liberzon (2008), the authors have proposed a quantized controller with minimal sensor data to achieve rendezvous of agents moving on a plane. However, most autonomous agents, like unmanned aerial vehicles (UAVs) and autonomous underwater vehicles (AUVs), more often move in 3D space. Though the work presented here is on the lines of Yu et al. (2008), the 3D problem has significant differences and complexity, in the sense that the geometry in 3D is more involved than the 2D case. Also, the extension of the notion of minimal sensing and coarse actuation to the 3D case is non-trivial.

The paper is organized as follows: Section 2 presents the vehicle model, the sensors and the control law. Section 3 presents the condition for rendezvous. Section 4 presents the simulation results for the problem. Section 5 concludes the paper.

2. Problem formulation

We assume n agents modeled similar to a UAV with simple kinematics. The agent can yaw and pitch with constant angular speed, and move in a straight path (cruise) with constant speed. The agent, however, cannot roll or slip laterally. Each agent has a conical field-of-view with infinite range within which it tries to maintain its target. The target for the ith agent is the agent $(i+1)$modulo n. The control is applied only when the target moves out of the windshield. We assume that all the agents have their target within their windshield initially. The vehicle model, sensors and control are now presented.

Vehicle model: Let $p_i = (x_i, y_i, z_i) \in \mathbb{R}^3$ be the position of the ith agent in an earth-fixed frame and $(\alpha_i, \beta_i, \gamma_i)$ be the Euler angles corresponding to the Z–Y–Z Euler angle convention for transformation from the body-fixed frame to the earth-fixed frame with anti-clockwise rotations. See Fig. 1. The forward (linear) velocity of the vehicle is only along its body X_b axis and its magnitude (v_i) remains constant. The vehicle can rotate about its body Y_b-axis (pitch) and about its body Z_b-axis (yaw) in both clockwise and counter-clockwise directions.

The pitch and yaw rates take values from the discrete set, $\omega_{y_{i_b}} \in \{-\omega_i, 0, +\omega_i\}$ and $\omega_{z_{i_b}} \in \{-\omega_i, 0, +\omega_i\}$, where ω_i is constant and $\omega_i > 0$. The agents are considered identical and hence $v_i = v_j$ and $\omega_i = \omega_j$.

Since the model involves rigid body motion (translation and rotation) in space, we relate the inertial velocity components to the body velocity components through the Euler angles as

$$[\dot{x}_i\ \dot{y}_i\ \dot{z}_i]^T = R_{z_{i_3}} R_{y_{i_2}} R_{z_{i_1}} [v_i\ 0\ 0]^T \tag{1}$$

where $(\dot{x}_i, \dot{y}_i, \dot{z}_i)$ are the velocity components in the inertial frame and $(v_i, 0, 0)$ are the velocity components in the body-fixed frame. $R_{z_{i_3}}, R_{y_{i_2}}, R_{z_{i_1}}$ are rotation matrices, locally parametrized by the Euler angles. The Euler angle rates are related to the vehicle body angular velocities as

$$[\dot{\alpha}_i\ \dot{\beta}_i\ \dot{\gamma}_i]^T = C(\alpha_i, \beta_i, \gamma_i)[0\ \omega_{y_{i_b}}\ \omega_{z_{i_b}}]^T \tag{2}$$

where $C(\alpha_i, \beta_i, \gamma_i)$ is the matrix corresponding to the angular rates for the Euler transformation. Eqs. (1) and (2) describe the kinematic model of the ith vehicle.

Sensors: The sensor for each agent has a conical view with the half angle of the cone being $\phi \in (0, \pi)$ as shown in Fig. 2(a). This field-of-view is termed as the *windshield*. The range of view within this angle is assumed to be infinite. It is also assumed that one agent cannot occlude the view of another agent when both appear within the windshield. The field-of-view, as seen by the agent, appears like a disc. This disc can be divided into four quadrants as shown in Fig. 2(b). The sensors do not give the actual distance between agents. They only give a discrete output based on the quadrant from which the assigned agent (the target) moves out.

Let the output set be O and the sensor measurement of the ith vehicle be $(o_{y_i}, o_{z_i}) \in O$. (o_{y_i}, o_{z_i}) takes the following values according to the quadrant from which the target agent j escapes from view:-

$$(o_{y_i}, o_{z_i}) = \begin{cases} (-\mathrm{sgn}(P_{y_b}), \mathrm{sgn}(P_{z_b})), & j \text{ escapes from } P \\ (0, 0), & \text{agent } j \text{ is in the view} \end{cases} \tag{3}$$

in which agent j is the target agent of i and P is the point of escape from the windshield. These values are also indicated in Fig. 2(b).

Controls: The output of the sensors actuates the controllers for necessary action. The control law is defined with respect to the sensor output as:

$$(\omega_{y_{i_b}}, \omega_{z_{i_b}}) = (o_{z_i}, o_{y_i})\omega_i. \tag{4}$$

As can be seen, this control law involves no history or state estimation. Next we try to find the conditions such that using this control law the agents achieve rendezvous.

3. Conditions for rendezvous

3.1. Concepts from graph theory

The agents are considered to be in cyclic pursuit. For simplicity and without loss of generality we assume that $(i+1)$modulo n is assigned to agent i. This system is represented by a digraph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ with the agents being its nodes $(i \in \mathscr{V})$ and $e_{i,i+1} \in \mathscr{E}(\mathscr{G})$. As agent i catches up with agent $i+1$, i and $i+1$ move as one entity $i+1$. This is called *merging*. The merging operation is triggered when $l_{i,i+1} \leq \rho$. The *merging radius*, $\rho(> 0)$, is the distance between the pursued and the pursuer after which they merge and move as one entity. Unlike many studies on this problem which consider a point mass geometry for the vehicles, we assume a finite geometry and hence a safe zone is considered surrounding each vehicle, which mathematically translates to the merging radius ρ. Merging occurs if $e_{i,i+1} \in \mathscr{E}(\mathscr{G})$, $l_{i,i+1} \leq \rho$. After merging, agent $i-1$ starts pursuing $i+1$. The node i is deleted and the edges $e_{i-1,i}$ and $e_{i,i+1}$ are deleted from $\mathscr{E}(\mathscr{G})$ and a new edge $e_{i-1,i+1}$ comes into effect. The number of nodes is reduced. The graph $\mathscr{G}$ is said to be *live* if it has at least one edge (Yu et al., 2008). A graph with a single vertex is also a live-graph. If $\mathscr{G}$ is not live then there exists more than one vertex with no edge.

3.2. A Lyapunov candidate

Let $V : \mathbb{R}^{3n} \to \mathbb{R}$ be a Lyapunov function which is defined as

$$V = \sum_{e_{i,j} \in \mathscr{E}(\mathscr{G})} l_{i,j}. \tag{5}$$

Since V is the sum of distances, it will always be positive and will go to zero only when the system achieves rendezvous. So V is termed as *rendezvous positive definite*. At each instant of merging V has a discontinuity. V is also called a *graph compatible Lyapunov function* as it is based on the digraph $\mathscr{G}$ (Yu et al., 2008). $\mathscr{G}$ is strongly connected and hence $\mathscr{G}$ is live for $t \geq 0$, and V is rendezvous positive definite (from Lemmas 1 and 2 Yu et al., 2008).