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Steady-state performance of multimodulus blind equalizers

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ABSTRACT

Multimodulus algorithms (MMA) based adaptive blind equalizers mitigate inter-symbol interference in a digital communication system by minimizing dispersion in the quadrature components of the equalized sequence in a decoupled manner, i.e., the in-phase and quadrature components of the equalized sequence are used to minimize dispersion in the respective components of the received signal. These unsupervised equalizers are mostly incorporated in bandwidth-efficient digital receivers (wired, wireless or optical) which rely on quadrature amplitude modulation based signaling. These equalizers are equipped with nonlinear error-functions in their update expressions which makes it a challenging task to evaluate analytically their steady-state performance. However, exploiting variance relation theorem, researchers have recently been able to report approximate expressions for steady-state excess mean square error (EMSE) of such equalizers for noiseless but interfering environment.

In this work, in contrast to existing results, we present exact steady-state tracking analysis of two multimodulus equalizers in a non-stationary environment. Specifically, we evaluate expressions for steady-state EMSE of two equalizers, namely the MMA2-2 and the βMMA. The accuracy of the derived analytical results is validated using different set experiments and found in close agreement.

1. Introduction

Transmission of signals between a transmitter and a receiver in a communication system encounters different types of dispersive channels. Such channels perform certain non-ideal transformations resulting in different types of interferences like inter-symbol interference (ISI) and frequency selective fading, which are considered to be the biggest limiting factors in a communication system. One of the approaches to combat ISI is to use blind equalizer. An adaptive blind equalizer attempts to compensate for the distortions of the channel by processing the received signals and reconstructing the transmitted signal up to some indeterminacies by the use of linear or nonlinear filters. Specifically, a blind equalizer does not require any training mode and tries to mitigate the effects of the channel solely on the basis of probabilistic and statistical properties of the transmitted data sequence. The basic idea behind an adaptive blind equalizer is to minimize or maximize some admissible blind objective or cost function through the choice of filter coefficients based on the equalizer output [1–3].

When an adaptive equalizer is used to combat a time-varying channel, the optimum Wiener solution takes time-varying form which results in variation of saddle

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point in error performance surface. If the underlying signal statistics happen to change with time, then these statistical variations will be reflected in the data, the filter has access to, which in turn will be reflected in the performance of filters. So, tracking variations in signal statistics or signal moments is considered to be a useful property for adaptive filters. For adaptive filters, the variation in underlying signal statistics and the saddle point can be tracked by using *tracking performance analysis*; and consequently, the filter parameters can be adjusted accordingly to maintain the saddle points of error performance surface and to calculate the variations in underlying signal statistics in time-varying systems. One metric to evaluate tracking performance of an adaptive filter is to measure the steady-state excess mean square error (EMSE). EMSE can be defined as the difference between the mean square error (MSE) of the filter in steady-state and the minimum cost. The smaller the EMSE of an adaptive filter, the better it is [4]. If filter parameters (like step-size) are chosen correctly, the filter can track variations in signal statistics provided variations are not fast. However, tracking fast variations in signal statistics might be a challenging task or at times impossible to perform [4].

In the context of adaptive blind equalization, the widely adopted algorithm is Constant Modulus Algorithm (CMA2-2) [2,5–7]. For quadrature amplitude modulation (QAM) signaling, however, a tailored version of CMA2-2, commonly known as Multimodulus Algorithm (MMA2-2), is considered to be more suitable. The MMA2-2 is capable of jointly achieving blind equalization and carrier phase recovery [8–13], whereas the CMA2-2 requires a separate phase-lock loop for carrier phase recovery.

The nonlinearity of most of the adaptive equalizers, including both CMA2-2 and MMA2-2, makes the steady-state analysis and tracking performance a difficult task to perform. As a result, only a small number of analyses are available in the literature concerning the steady-state analysis performance of adaptive equalizers. However, a few results are available on EMSE analysis of CMA2-2, where some researchers employed Lyapunov stability and averaging analysis [14], and some exploited the variance relation theorem [15,16] to evaluate the same. The steady-state analysis of adaptive filters has gained interest due to ease in analysis owing to variance relation theorem. Abrar et al. [17] performed the EMSE analysis of CMA2-2 and βCMA [18] by assuming that the modulus of equalized signals is Rician distributed in the steady-state. Moreover, this theorem has been employed to study the steady-state analysis of a number of adaptive blind equalization algorithms like in the analyses of the so-called hybrid algorithm [19], the square contour algorithm [20], the improved square contour algorithm [21] and the varying-modulus algorithms [22].

In this paper, we perform tracking performance analysis of two well-known multimodulus equalizers. In particular, using the variance relation arguments, we derive expressions for steady-state EMSE of MMA2-2 and recently proposed βMMA [23] under the assumption that the quadrature components of the successfully equalized signal are Gaussian distributed when conditioned on true signal alphabets. The paper is organized as follows: Section 2 introduces the mathematical model for the system. Section 3 introduces the non-stationary environment and the framework for EMSE analyses. Section 3.1 provides the steady-state tracking performance analysis for MMA2-2 equalizer. Section 3.2 presents the analytical expression evaluated for steady-state tracking performance analysis for βMMA equalizer. Section 4 provides simulation results for steady-state performances of MMA2-2 and βMMA for equalized zero-forcing scenario, equalization of fixed and time-varying channels, and equalization under different values of filter-length. Finally, Section 5 draws conclusions.

2. System model and multimodulus equalizers

A typical baseband communication system is given in Fig. 1. Consider the transmission of discrete valued complex sequence $\{a_n\}$ over an unknown communication channel characterized by finite impulse response filter with impulse response $\boldsymbol{h}_n$; the sequence $\{a_n\}$ is independent and identically distributed (i.i.d.), and takes value of square-QAM symbols with equal probability. The considered channel $\boldsymbol{h}_n$ is a fading, dispersive, time-varying in nature, where the channel at index n is given as $\boldsymbol{h}_n = \boldsymbol{h}_{\text{const}} + \boldsymbol{c}_n$. The channel is a complex Gaussian random process with a constant mean $\boldsymbol{h}_{\text{const}}$ (because of shadowing, reflections and large scale path loss) and a time-variant part $\boldsymbol{c}_n$, the channel taps vary from symbol to symbol and are modeled as mutually uncorrelated circular complex Gaussian random processes. The time-varying part of the channel can be modeled by a pth-order autoregressive process AR(p).

The received signal x_n is the convolution of transmitted sequence $\{a_n\}$ and filter impulse response $\boldsymbol{h}_n$ represented as $x_n = \boldsymbol{h}_n^T \boldsymbol{a}_n$, where superscript T denotes the transpose operator. The vector $\boldsymbol{x}_n$ is fed to the equalizer to combat the interference introduced by the physical channel and estimate delayed version of the transmitted sequence $\{a_{n-\delta}\}$, where δ denotes delay.

Let $\boldsymbol{w}_n = [w_{n,0}, w_{n,1}, \ldots, w_{n,N-1}]^T$ be the impulse response of equalizer and $\boldsymbol{x}_n = [x_n, x_{n-1}, \ldots, x_{n-N+1}]^T$ be the regression vector (vector of channel observations), N is the number of equalizer taps. The output of equalizer is convolution of regression vector and equalizer impulse response given as $y_n = \boldsymbol{w}_{n-1}^H \boldsymbol{x}_n$ where superscript H denotes the Hermitian conjugate operator. Let $\boldsymbol{t}_n = \boldsymbol{h}_n \otimes \boldsymbol{w}_{n-1}^*$ be the overall channel-equalizer impulse response ($\otimes$ denotes convolution operation and the superscript $*$ denotes complex conjugate operator). If the channel response is given by a K-tap vector $\boldsymbol{h}_n = [h_{n,0}, h_{n,1}, \ldots,$

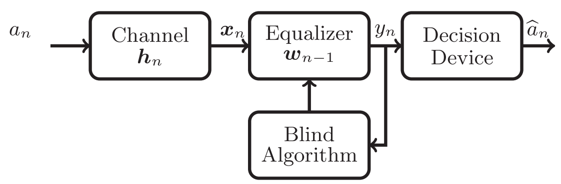

Fig. 1. A typical baseband communication system.