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Reprint of: Chaos synchronization of the discrete fractional logistic map ☆

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ABSTRACT

In this paper, master–slave synchronization for the fractional difference equation is studied with a nonlinear coupling method. The numerical simulation shows that the designed synchronization method can effectively synchronize the fractional logistic map. The Caputo-like delta derivative is adopted as the difference operator.

1. Introduction

Compared with the fruitful results in the chaos synchronization of continuous fractional differential equations, the fractional difference equation is a particularly new topic. The fractional difference has a short history. A pioneering work has been done by Miller and Ross [1] on discretization of the fractional calculus, and also benefiting from the theory of time scales originated by Hilger [2], several authors developed the theory of the discrete fractional calculus (DFC), such as the initial value problems [3], the discrete calculus of variations [4], the Laplace transform [5], signal processing based on the discrete-time derivatives [6,7] and so on [8–18].

The discrete version of the fractional calculus is a new direction which has huge potential applications in many areas of science and engineering. The application has gained much attention during the last few years. The aim of the recent studies in this area is twofold. On the one hand, the fractional difference equations generalize the classical difference equations. On the other, they make a possible comparison between the behaviors of the fractional difference and the fractional differential equations.

The dynamical behaviors of the fractional one and two dimensional maps and the results show that the chaos does exist there [19–21]. The DFC is proved to be an efficient tool to discrete the chaotic systems with memory effects.

Naturally, a question may be put forth: whether there is discrete fractional synchronization of such maps? In this

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paper, we investigate the chaos synchronization of the discrete fractional logistic map in [19] and design the synchronized systems.

2. Preliminaries

Assume that $\mathbb{N}_a$ denotes the isolated time scale and $\mathbb{N}_a=\{a,a+1,a+2,\ldots\}$ ($a\in\mathbb{R}$ fixed). For the function $f(n)$, the difference operator Δ is defined as $\Delta f(n)=f(n+1)-f(n)$.

Definition 2.1 (*Atici and Eloe* [3]). Let $x:\mathbb{N}_a\to\mathbb{R}$ and $0<\nu$ be given. Then the fractional sum of ν order is defined by

$$\Delta_a^{-\nu}x(t):=\frac{1}{\Gamma(\nu)}\sum_{s=a}^{t-\nu}(t-\sigma(s))^{(\nu-1)}x(s),\quad t\in\mathbb{N}_{a+\nu} \tag{1}$$

where a is the starting point, $\sigma(s)=s+1$ and $t^{(\nu)}$ is the falling function defined as

$$t^{(\nu)}=\frac{\Gamma(t+1)}{\Gamma(t+1-\nu)}. \tag{2}$$

Definition 2.2 (*Abdeljawad* [13]). For $0<\nu<1$, and $x(t)$ defined on $\mathbb{N}_a$, the Caputo-like delta difference is defined by

$${}^C\Delta_a^{\nu}x(t):=\frac{1}{\Gamma(1-\nu)}\sum_{s=a}^{t-(1-\nu)}(t-\sigma(s))^{(-\nu)}\Delta x(s),\quad t\in\mathbb{N}_{a+1-\nu}. \tag{3}$$

Now using the DFC, we can derive the discrete version of the logistic equation as [19]

$${}^C\Delta_a^{\nu}x(t)=\mu x(t+\nu-1)(1-x(t+\nu-1)),\quad t\in\mathbb{N}_{a+1-\nu},\quad 0<\nu<1. \tag{4}$$

Theorem 2.3 (*Chen et al.* [12]). *The delta fractional difference equation.* (4) *has the equivalent discrete integral equation*

$$x(t)=x_0+\frac{\mu}{\Gamma(\nu)}\sum_{s=a+1-\nu}^{t-\nu}(t-\sigma(s))^{(\nu-1)}x(s+\nu-1)(1-x(s+\nu-1)), \tag{5}$$

where $x_0=x(a), t\in\mathbb{N}_{a+1}$. *In this section we briefly present some basic definitions of the fractional difference operators. More details on this object can be found in* [13] *and the references therein.*

3. Chaos synchronization of the fractional logistic map

Pecora and Carroll [22] first presented chaos synchronization results for possibly chaotic dynamical systems. They reported that two identical systems can be coupled in some way so that the solution of one always converges to the other one independently of the initial conditions. Since then, much attention has been attracted from synchronization phenomena such as phase synchronization [23], small world network [24] and the fractional system [25].

In this paper, we consider Eq. (4) as a master system and suggest the coupled slave logistic map into

$${}^C\Delta_a^{\nu}y(t)=\mu y(t+\nu-1)(1-y(t+\nu-1))+H(x(t+\nu-1),\; y(t+\nu-1)),\quad y(a)=y_0 \tag{6}$$

where $t\in\mathbb{N}_{a+1-\nu}$, $H(x(t),y(t))=K(x(t)-y(t))-P(x^2(t)-y^2(t))$, and K and P are the linear and nonlinear coupling strengths, respectively.

As is well known, if we have

$$\lim_{t\to\infty}|x(t)-y(t)|=0 \tag{7}$$

we call (4) and (6) synchronized systems.

From Theorem 2.3, we can obtain the following equivalent discrete integral forms for the master and slave systems:

$$x(t)=x(a)+\frac{\mu}{\Gamma(\nu)}\sum_{s=a+1-\nu}^{t-\nu}(t-s-1)^{(\nu-1)}x(s+\nu-1)(1-x(s+\nu-1)) \tag{8}$$

and

$$y(t)=y(a)+\frac{\mu}{\Gamma(\nu)}\sum_{s=a+1-\nu}^{t-\nu}(t-s-1)^{(\nu-1)}[y(s+\nu-1)\times(1-y(s+\nu-1))+H] \tag{9}$$

where $t\in\mathbb{N}_{a+1}$ and $H=H(x(s+\nu-1),y(s+\nu-1))$.

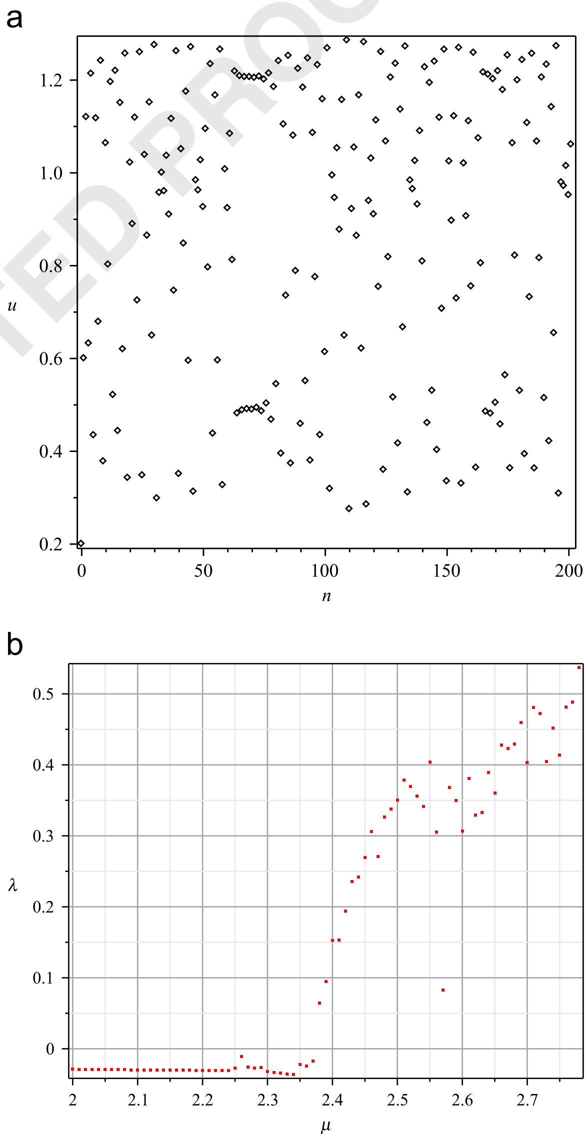

Fig. 1. The chaotical statue distinguished by the positive Lyapunov exponent. (a) Chaos for $\mu=2.5$ and $\nu=0.8$. (b) The Lyapunov exponent.

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