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Brief paper

Performance limitations for single-input LTI plants controlled over SNR constrained channels with feedback[☆]

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ABSTRACT

This paper studies control problems for discrete-time single-input linear time-invariant plants when controlled over a signal-to-noise ratio (SNR) constrained channel. Our focus is on the performance limitations in an architecture that uses channel feedback. We explicitly characterize the interplay between stabilization, optimal performance, and SNR constraints, highlighting the way in which plant dynamical features affect the best achievable performance. We also apply our results to the study of networked control systems where communication takes place over a power constrained erasure channel. In that scenario, we first show that stabilization problems, and problems involving stationary second-order moments, can be dealt with by focusing on a related SNR constrained networked situation. This observation allows one to obtain results valid in the alternative scenario as corollaries of the results obtained when a single SNR constraint is present.

1. Introduction

Control systems where communication takes place over non-transparent communication links are called networked control systems (NCSs) (Antsaklis & Baillieul, 2007). The study of NCSs has spawned a number of results on the interplay between communication constraints and control objectives (see, e.g., Antsaklis & Baillieul, 2007; Charalambous & Farhadi, 2008; Freudenberg, Middleton, & Braslavsky, 2011; Martins & Dahleh, 2008; Matveev & Savkin, 2009; Minero, Franceschetti, Dey, & Nair, 2009; Nair & Evans, 2004; Nair, Fagnani, Zampieri, & Evans, 2007; Sahai & Mitter, 2006; Schenato, Sinopoli, Franceschetti, Poolla, & Sastry, 2007; Tatikonda, Sahai, & Mitter, 2004, and the references therein). Existing results can be broadly classified into two groups: A first group of works have adopted an information theoretic approach (see, e.g., Charalambous & Farhadi, 2008; Martins & Dahleh, 2008; Matveev & Savkin, 2009; Nair & Evans, 2004; Sahai & Mitter, 2006; Tatikonda et al., 2004), establishing results valid in networked control situations which are analogues to fundamental results in information theory (Cover & Thomas, 2006). An advantage of such an approach is its generality. Indeed, most of the times, channels are characterized by means of an abstract probabilistic description and, hence, results in Charalambous and Farhadi (2008), Matveev and Savkin (2009), Sahai and Mitter (2006) and Tatikonda et al. (2004) are valid in broad scenarios. Another set of works focuses on specific communication constraints and uses techniques closely related to traditional control theory. For instance, Schenato et al. (2007) studies the effects of data-dropouts, Braslavsky, Middleton, and Freudenberg (2007) and Freudenberg et al. (2011) studies power (or signal-to-noise ratio (SNR)) constraints, and Nair et al. (2007) addresses data-rate constraints. A good survey of this line of work is Hespanha, Naghshtabrizi, and Xu (2007). More recent work includes Freudenberg et al. (2011), Minero et al. (2009) and You and Xie (2011).

In this paper, we adopt a simplified approach and consider feedback control problems subject to SNR constraints (Braslavsky et al., 2007). We believe that such a framework is relevant for several reasons. First, the SNR approach uses linear time-invariant (LTI) control theory and, thus, enables one to use a wealth of well-known synthesis and analysis techniques. Second, conclusions derived for SNR constrained NCSs can be immediately translated into conclusions valid in more interesting scenarios, including control subject to average data-rate constraints or data-dropouts (Silva, Derpich, & Østergaard, 2011b; Silva & Pulgar, 2011a).

The study of SNR constrained NCSs was started in Braslavsky et al. (2007) (see also Braslavsky, Middleton, & Freudenberg, 2004). That work studies stabilization problems over power constrained additive noise channels. For the discrete-time case, Braslavsky et al. (2007) shows that static state feedback controllers allow one to stabilize a plant, while satisfying the channel input power

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constraint, if and only if the channel SNR is greater than a simple function of the unstable plant poles. In the dynamic output feedback case, it is shown in Braslavsky et al. (2007) that the minimal SNR compatible with stability is also a function of the non-minimum phase zeros and the relative degree of the plant. The results of Braslavsky et al. (2007) have been extended in, e.g., Middleton, Rojas, Freudenberg, and Braslavsky (2009), Rojas, Braslavsky, and Middleton (2008b) and Silva, Goodwin, and Quevedo (2010).

The above references focus on stabilization problems only. As such, the derived controllers, whilst optimal from the point of view of achieving stability at the lowest possible channel SNR, may be unsuitable for disturbance compensation or robust stabilization (Freudenberg, Braslavsky, & Middleton, 2005). Having this issue in mind, Freudenberg et al. (2005) presents an approach for improving the properties of the solutions to stabilization problems. In a related work, Rojas, Braslavsky, and Middleton (2008a) characterizes the additional SNR incurred when choosing an arbitrary sensitivity function, but does not characterize optimal ones.

The references in the previous two paragraphs do not explicitly deal with optimal control problems. A first indication of the tradeoffs between SNR constraints and achievable performance is presented in Freudenberg, Middleton, and Solo (2010). That work studies NCSs where the plant state is measured without noise, and nonlinear time varying pre- and post-processing is used around an additive noise channel. It is shown in Freudenberg et al. (2010) that the condition derived in Braslavsky et al. (2007) for state feedback problems remains necessary for mean square stabilization in this case. It is also shown in Freudenberg et al. (2010) that, if the channel SNR is close to the minimum SNR for stabilization, then the norm of the state will become arbitrarily large in the presence of disturbances. This conclusion is consistent with similar results in Nair et al. (2007).

Optimal design problems are addressed in Freudenberg, Middleton, and Braslavsky (2007). In that work, the authors characterize the minimal plant output variance in a one degree-of-freedom control architecture closed over a power constrained additive noise channel. The results in Freudenberg et al. (2007) are presented for minimum phase plants that have relative degree one, and several structural properties of the solution are discussed. An alternative approach to optimal control over Gaussian channels is proposed in Freudenberg et al. (2011). In that work, the authors focus on plant output variance minimization at a given terminal time and derive linear time-varying control and communication strategies that are shown, in some cases, to be universally optimal (i.e., optimal within the class of nonlinear time-varying schemes). A difficulty with the results of Freudenberg et al. (2011) is that optimizing performance at a given terminal instant usually yields poor transient behaviour.

In this paper, we study performance limitations for discrete-time single-input LTI plants when controlled over an SNR constrained channel. By a performance limitation we mean a closed-form characterization of the best achievable performance for a given networked situation. When studying performance limitations, one strives for clarity and insight instead of full generality (see, e.g., papers in Chen & Middleton, 2003). Works presenting performance limitations subject to SNR constraints include Ding, Wang, Guan, and Chen (2010), Freudenberg et al. (2007), Guan, Zhan, and Feng (2011) and Li, Tuncel, Chen, and Su (2009). Those papers focus on situations where the controller is placed at the receiving end of the channel. In particular, Freudenberg et al. (2007) and Li et al. (2009) consider minimum-phase plants and give a simple characterization of the best achievable performance in terms of plant dynamical features. Related results are presented in Ding et al. (2010) and Guan et al. (2011) for more general plants, under the assumption that infinite channel input power is available. The above results should be contrasted with the numerical approach for solving SNR constrained control problems proposed in Silva et al. (2010).

This paper considers a control architecture that uses channel feedback and where the controller is located at the sending end of the channel. Our main contribution is an explicit characterization of the interplay between stabilization, optimal performance, and SNR constraints in such an architecture. Our results are given in closed-form and highlight the way in which plant dynamical features, namely unstable plant poles, non-minimum phase zeros, and plant frequency response, affect the best achievable performance.

To emphasize the relevance of our results, we apply them to the study of NCSs where communication takes place over a power constrained erasure channel. Such a channel combines two sources of communication constraints: data-dropouts and input power constraints. As a second contribution, we exploit the results in Silva and Pulgar (2011a) to show that, as far as stationary second-order statistics are concerned, control problems over a power constrained erasure channel are equivalent to control problems subject to a single SNR constraint. This result allows one to obtain results valid in the alternative erasure channel scenario, as direct consequences of the results derived for a single SNR constraint. Extensions to control problems over digital erasure channels are reported in Silva and Pulgar (2011b).

The problems addressed in this paper can be seen as special cases of the problems studied in, e.g., Charalambous and Farhadi (2008), Matveev and Savkin (2009), Sahai and Mitter (2006) and Tatikonda et al. (2004). However, the results in those references are not well suited for the study of explicit performance limitations in the spirit of this paper.

The remainder of the paper is organized as follows: Section 2 describes the setup. Section 3 presents results pertaining to SNR constrained NCSs. Section 4 addresses problems over power constrained erasure channels. Section 5 draws conclusions.

Notation. $\triangleq$ stands for "defined as". $\mathbb{R}^+ \triangleq \{x \in \mathbb{R} : 0 < x < \infty\}$; $|x|$ and $\bar{x}$ stand for the magnitude and conjugate of the complex number x; X^H and X^T denote the conjugate transpose and transpose of the matrix X; $\ln(\cdot)$ refers to natural logarithm; I_n refers to the $n \times n$ identity matrix. We work in discrete time, and use z as the forward shift operator and also as the argument of the Z-transform. If $X(z)$ is a real rational transfer function, then we usually omit the dependence on z and simply write X. The set of all proper (resp. strictly proper) real-rational discrete-time transfer functions of dimension $n \times m$ is denoted by $\mathcal{R}_p^{n\times m}$ (resp. $\mathcal{R}_{sp}^{n\times m}$); $\mathcal{RH}_\infty^{n\times m}$ denotes the set of all stable and proper discrete-time real-rational transfer functions of dimension $n \times m$. If x is an (asymptotically) wide sense stationary process, then σ_x^2 denotes its stationary variance (i.e., the trace of the corresponding stationary covariance matrix). The space $\mathcal{L}_2$ is defined as usual, and its norm (the 2-norm) is denoted by $\|\cdot\|_2$ (Zhou, Doyle, & Glover, 1996).

2. Problem setup

We consider the control scheme depicted in Fig. 1. In that figure, G is a discrete-time single-input multiple-output (SIMO) LTI plant, K is a discrete-time multiple-input single-output LTI controller, d is a disturbance, and the channel is an additive white noise channel with feedback. The controller in Fig. 1 is allowed to use the plant output y, and (one-step delayed) channel feedback $\hat{u}$, to construct the channel input v.

Assumption 1. (a) The plant is nonzero, free of unstable hidden modes, and its transfer function G belongs to $\mathcal{R}_{sp}^{n\times 1}$.

(b) The disturbance d is a second-order zero-mean white noise sequence having variance $\sigma_d^2 \in \mathbb{R}^+$. The joint initial state of G and K, say x_o, is a second-order random variable.