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Brief paper

When retarded nonlinear time-delay systems admit an input–output representation of neutral type[☆]

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ABSTRACT

The paper shows that nonlinear retarded time-delay systems can admit an input–output representation of neutral type. This behaviour represents a strictly nonlinear phenomenon, for it cannot happen in the linear time-delay case where retarded systems always admit an input–output representation of retarded type. A necessary and sufficient condition for a nonlinear system to exhibit this behaviour is given, and a strategy for finding an input–output representation of retarded type is outlined. Some open problems that arise consequently are discussed as well. All the systems considered in this work are time-invariant and have commensurable delays.

1. Introduction

In control theory, systems are usually described either by a set of coupled first-order differential equations, called state-space representation, or by higher order input–output differential equations. In the linear case any control system described by the state-space equations can equivalently be described by higher order input–output differential equations. From that point of view Laplace transforms play a key role. In the nonlinear case the situation is more complicated, and several techniques have been developed to find the corresponding input–output equations; see for instance Conte, Moog, and Perdon (2007), and Diop (1991). Considering the algebraic point of view, it was shown by Conte et al. (2007) that for a given state-space representation a corresponding set of input–output equations can be, at least locally, always constructed by applying a suitable change of coordinates. Such an idea of the state elimination has recently been carried over by Anguelova and Wennberg (2008) to nonlinear time-delay systems, and it has been shown that even for a state-space system with delays there always exists, at least locally, a set of input–output differential-delay equations. Note that the systems under consideration are time-invariant and have commensurable delays. However, the state elimination of Anguelova and Wennberg (2008) might result in a set of input–output equations representing a system of neutral type, even when one starts with the state-space equations being of retarded type. This can also be suspected from the inversion algorithm of Márquez-Martínez, Moog, and Velasco-Villa (2000). Note that by retarded we mean a classical (non-neutral) system, and by neutral a system having delays in the highest derivative. Such a behaviour represents a strictly nonlinear phenomenon, for it cannot happen in the linear time-delay case where retarded systems always admit an input–output representation of a retarded type, and forms the main scope of our interest in this paper. In particular, we show why it cannot happen in the linear time-delay case, and why and when it happens in the nonlinear time-delay case.

The paper is organized as follows. In Section 2 an algebraic background for dealing with nonlinear time-delay systems is briefly recalled. The state elimination, and how it might result in an input–output equation of neutral type, is discussed in Section 3, and followed by the main results in Section 4 where we show why this cannot happen in the linear time-delay case, and why and when it happens in the nonlinear time-delay case. Then, a strategy for obtaining an input–output representation of retarded type is

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suggested in Section 5. Finally, conclusions and open problems are discussed in Section 6.

2. Algebraic setting

In this paper we use the mathematical setting of Anguelova and Wennberg (2008); Márquez-Martínez, Moog, and Velasco-Villa (2002); Moog, Castro-Linares, Velasco-Villa, and Márquez-Martínez (2000) and Xia, Márquez-Martínez, Zagalak, and Moog (2002).

Consider nonlinear time-delay systems of the form

$$\begin{aligned} \dot{x}(t) &= f(\{x(t-i), u(t-j); i, j \geq 0\}) \\ y(t) &= h(\{x(t-i); i \geq 0\}) \end{aligned} \tag{1}$$

where the entries of f and h are meromorphic functions, and $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}$ denote state, input and output to the system respectively.

Remark 1. Assuming the system has commensurable delays, it is not restrictive to consider $i, j \in \mathbb{N}$ since all the delays can be treated as multiples of an elementary delay τ.

Denote by $i_{\max}$ the maximal delay in (1). The function of initial conditions $\varphi : [-i_{\max}, 0] \to \mathbb{R}^n$ is assumed to be smooth on an open interval containing $[-i_{\max}, 0]$. The input (control variable) $u : [-i_{\max}, \infty) \to \mathbb{R}^m$ is smooth for $t > -i_{\max}$. For a given φ, the set of inputs $u(t)$ for which there exists a unique solution to system (1) in the interval $[0, \infty)$ are called admissible inputs. Let $C \subset C^\infty$ denote the open set of φ with a non-empty set of admissible inputs.

Let $\mathcal{K}$ be the field of meromorphic functions of $\{x(t-i), u^{(k)}(t-j); i, j, k \geq 0\}$, and $\mathcal{E} = \mathrm{span}_{\mathcal{K}}\{\mathrm{d}\xi; \xi \in \mathcal{K}\}$ the formal vector space of differential one-forms. Let δ denote the delay operator defined as

$$\begin{aligned} \delta(\xi(t)) &= \xi(t-1) \\ \delta(\alpha(t)\mathrm{d}\xi(t)) &= \alpha(t-1)\mathrm{d}\xi(t-1) \end{aligned} \tag{2}$$

for any $\xi(t) \in \mathcal{K}$ and $\alpha(t)\mathrm{d}\xi(t) \in \mathcal{E}$.

The delay operator (2) induces the (non-commutative) skew polynomial ring $\mathcal{K}(\delta]$ with the usual addition and (non-commutative) multiplication given by the commutation rule

$$\delta a(t) = a(t-1)\delta$$

for any $a(t) \in \mathcal{K}$. The ring $\mathcal{K}(\delta]$ is Noetherian and a left Ore domain.

Lemma 2 (*Ore Condition*)**.** *For all nonzero $a(\delta], b(\delta] \in \mathcal{K}(\delta]$ there exist nonzero $a_1(\delta], b_1(\delta] \in \mathcal{K}(\delta]$ such that $a_1(\delta]b(\delta] = b_1(\delta]a(\delta]$.*

The properties of system (1) can now be analysed by introducing the formal module $\mathcal{M} = \mathrm{span}_{\mathcal{K}(\delta]}\{\mathrm{d}\xi; \xi \in \mathcal{K}\}$. The rank of a module over the left Ore domain $\mathcal{K}(\delta]$ is well-defined (Cohn, 1985).

Definition 3 (*Xia et al., 2002*)**.** The closure of a submodule $\mathcal{N}$ in $\mathcal{M}$ is the submodule

$$\overline{\mathcal{N}} = \{w \in \mathcal{M}; \exists a(\delta] \in \mathcal{K}(\delta], a(\delta]w \in \mathcal{N}\}.$$

That is, it is the largest submodule of $\mathcal{M}$ containing $\mathcal{N}$ with rank equal to $\mathrm{rank}_{\mathcal{K}(\delta]}\mathcal{N}$.

The notion of the closure of a submodule will play a key role in showing why and when the systems of the form (1) admit an input–output equation of a neutral type.

3. Input–output representation

3.1. State elimination

In this subsection we recall the state elimination of Anguelova and Wennberg (2008). For the sake of simplicity we sometimes use the notation $\psi(\delta, z_1, \ldots, z_k) := \psi(z_1(t), \ldots, z_1(t-i_1), \ldots, z_k(t-i_k), \ldots, z_k(t-i_k))$ for ψ and $z_1, \ldots, z_k$ in $\mathcal{K}$ with $i_1, \ldots, i_k$ nonnegative.

Let f be an r-dimensional vector with entries $f_j \in \mathcal{K}$. Let $\partial f/\partial x$ denote the $r \times n$ matrix with entries

$$\left(\frac{\partial f}{\partial x}\right)_{j,i} = \sum_{\ell \geq 0} \frac{\partial f_j}{\partial x_i(t-\ell)}\delta^\ell \in \mathcal{K}(\delta].$$

Denote by s the least nonnegative integer such that

$$\mathrm{rank}_{\mathcal{K}(\delta]}\frac{\partial(h, \ldots, h^{(s-1)})}{\partial x} = \mathrm{rank}_{\mathcal{K}(\delta]}\frac{\partial(h, \ldots, h^{(s)})}{\partial x}. \tag{3}$$

The integer s is called an observability index. Let $S = (h, \ldots, h^{(s-1)})$ then $\mathrm{rank}_{\mathcal{K}(\delta]}\partial S/\partial x = s \leq n$. Hence

$$\frac{\partial h^{(s)}}{\partial x} \in \overline{\mathrm{span}_{\mathcal{K}(\delta]}\left\{\frac{\partial(h, \ldots, h^{(s-1)})}{\partial x}\right\}}.$$

Thus, there exists a nonzero polynomial $b(\delta] \in \mathcal{K}(\delta]$ such that

$$b(\delta]\frac{\partial h^{(s)}}{\partial x} \in \mathrm{span}_{\mathcal{K}(\delta]}\left\{\frac{\partial(h, \ldots, h^{(s-1)})}{\partial x}\right\}. \tag{4}$$

That is

$$b(\delta]\frac{\partial h^{(s)}}{\partial x} = \sum_{j=0}^{s-1} b_j(\delta]\frac{\partial h^{(j)}}{\partial x} \tag{5}$$

for some $b_j(\delta] \in \mathcal{K}(\delta], j = 0, \ldots, s-1$. Therefore

$$b(\delta]\mathrm{d}h^{(s)} + \sum_{r=1}^{m}\sum_{j=0}^{J} c_{j,r}(\delta]\mathrm{d}u_r^{(j)} - \sum_{j=0}^{s-1} b_j(\delta]\mathrm{d}h^{(j)} = 0$$

for some $J \geq 0$, where J is the highest derivative of u appearing in the functions in S and $c_{j,r}(\delta] \in \mathcal{K}(\delta]$. We assume that the polynomials $b(\delta]$, $b_j(\delta]$ and $c_{j,r}(\delta]$ have no common factors other than 1. Since all functions are assumed meromorphic, and we have continuous dependence for the output on the input and initial function, the above equality holds on an open and dense subset of C. Applying the Poincaré lemma, we obtain a function $\xi(t) \in \mathcal{K}$ such that

$$\mathrm{d}\xi = b(\delta]\mathrm{d}h^{(s)} + \sum_{r=1}^{m}\sum_{j=0}^{J} c_{j,r}(\delta]\mathrm{d}u_r^{(j)} - \sum_{j=0}^{s-1} b_j(\delta]\mathrm{d}h^{(j)} \tag{6}$$

and

$$\xi\left(\delta, y, \ldots, y^{(s)}, u, \ldots, u^{(J)}\right) = 0. \tag{7}$$

We have obtained an input–output representation for the system (1).

3.2. Neutral input–output equations

The state elimination can result in an input–output equation representing a system of neutral type, even if one starts with a retarded system.

Definition 4. The input–output representation (7) of the system (1) is said to be neutral, if

$$\frac{\partial \xi(\cdot)}{\partial y^{(s)}(t-i)} \neq 0$$

for some $i \geq 1$.

Simple systems can generate this phenomenon, as shown in the following examples.