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Brief paper

Lyapunov-based hybrid loops for stability and performance of continuous-time control systems[☆]

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ABSTRACT

We construct hybrid loops that augment continuous-time control systems. We consider a continuous-time nonlinear plant in feedback with a (possibly non stabilizing) given nonlinear dynamic continuous-time state feedback controller. The arising hybrid closed loops are guaranteed to follow the underlying continuous-time closed-loop dynamics when flowing and to jump in suitable regions of the closed-loop state space to guarantee that a positive definite function V of the closed-loop state and/or a positive definite function V_p of the plant-only state is non-increasing along the hybrid trajectories. Sufficient conditions for the construction of these hybrid loops are given for the nonlinear case and then specialized for the linear case with the use of quadratic functions. For the linear case we illustrate specific choices of the functions V and V_p which allow for the reduction of the overshoot of a scalar output. The proposed approaches are illustrated on linear and nonlinear examples.

1. Introduction

For a large class of nonlinear control systems which follow a purely continuous dynamics, it may be useful to consider dynamic controllers having a mixed discrete/continuous dynamics. This leads to the class of hybrid control laws which has been proven to relax certain limitations of continuous-time controllers. Among other things, hybrid controllers are also instrumental to improve the performance for nonlinear systems in the presence of disturbances. See Prieur and Astolfi (2003) for the non-holonomic integrator, and Sanfelice, Teel, Goebel, and Prieur (2006) for the inverted pendulum to focus on applications only. Also for linear plants, hybrid controllers can be fruitful. See Beker, Hollot, and Chait (2001) for an example of a reset controller overcoming intrinsic limitations of linear control schemes. See also Beker, Hollot, and Chait (2004); Nešić, Zaccarian, and Teel (2008) where reset controllers are used to decrease the $\mathcal{L}_2$ gain between perturbations and the output. Consider also Aangenent, Witvoet, Heemels, van deMolengraft, and Steinbuch (2010) where it is shown that reset controllers may be useful to improve the $\mathcal{L}_2$ or $\mathcal{H}_2$ stability of linear systems. Finally, see Lazar and Heemels (2009) for the design of predictive controllers for the input-to-state stability of hybrid systems.

In this paper we consider a nonlinear plant:

$$\dot{x}_p = \bar{f}_p(x_p, u), \tag{1}$$

with x_p in $\mathbb{R}^{n_p}$, in feedback interconnection with a (not necessarily stabilizing) dynamic controller:

$$\dot{x}_c = \bar{f}_c(x_c, x_p), \qquad u = \bar{h}_c(x_c, x_p), \tag{2}$$

with x_c in $\mathbb{R}^{n_c}$. Then defining the closed-loop functions $f_p(x_p, x_c) = \bar{f}_p(x_p, \bar{h}_c(x_c, x_p))$ and $f_c(x_c, x_p) = \bar{f}_c(x_c, x_p)$, the interconnection between (1) and (2) can be described in a compact way as:

$$\frac{d}{dt}(x_p, x_c) = (f_p(x_p, x_c), f_c(x_c, x_p)), \tag{3}$$

where $f_p : \mathbb{R}^{n_p} \times \mathbb{R}^{n_c} \to \mathbb{R}^{n_p}$ and $f_c : \mathbb{R}^{n_c} \times \mathbb{R}^{n_p} \to \mathbb{R}^{n_c}$. We will assume that $\bar{f}_p$, $\bar{f}_c$ and $\bar{h}_c$ are such that f_p and f_c are continuous functions satisfying $f_p(0, 0) = 0$ and $f_c(0, 0) = 0$.

The contribution of this paper is to design a suitable reset rule, or jump law, for controller (2), and to design a partition of

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the state space $\mathbb{R}^n$ (where $n = n_p + n_c$) in two subsets, called flow and jump sets. The state x_c of controller (2) endowed with such additional logic, is then instantaneously reset according to the jump law whenever the state belongs to the jump set. This extended scheme, which is allowed to flow according to (3) only if the state belongs to the flow set, defines a hybrid system. More specifically, the proposed hybrid augmentation is designed to guarantee the decrease of one or both of two scalar Lyapunov-like functions, one of them $V : \mathbb{R}^{n_p} \times \mathbb{R}^{n_c} \to \mathbb{R}_{\geq 0}$, defined on the whole state space, and the other one $V_p : \mathbb{R}^{n_p} \to \mathbb{R}_{\geq 0}$ defined only in the plant state subspace. The functions V and V_p can be selected to capture some closed-loop stability and performance property (see the developments in Section 3.2 where output overshoot reduction is tackled). Functions V and V_p are linked to each other by a function $\phi : \mathbb{R}^{n_p} \to \mathbb{R}^{n_c}$ such that for all (x_p, x_c) in $\mathbb{R}^{n_p} \times \mathbb{R}^{n_c}$

$$V(x_p, \phi(x_p)) \leq V(x_p, x_c), \tag{4}$$

and, in particular, by the relation

$$V_p(x_p) = V(x_p, \phi(x_p)), \quad \forall x_p \in \mathbb{R}^{n_p}. \tag{5}$$

Within the above scenario, we will design flow and jump sets and jump rules such that the arising hybrid systems guarantee non-increase of V or V_p, or both of them in two relevant cases:

- (V) (addressed in Section 2.1) where the function $V(\cdot,\cdot)$ is given and satisfies suitable conditions guaranteeing the existence[2] of $\phi(\cdot)$ from which $V_p(\cdot)$ can be derived according to (5);
- (V_p) (addressed in Section 2.2) where $V_p(\cdot)$ and $\phi(\cdot)$ are given (their existence resembles an asymptotic controllability assumption), from which $V(\cdot,\cdot)$ satisfying (4) and (5) will be constructed.

Section 3 deals with the special case where system (3) is linear. In this case, quadratic versions of V and V_p can be constructed under reasonably weak properties required for the closed-loop dynamics. The extension to the linear case allows to strengthen the nonlinear results by exploiting the homogeneity property of hybrid systems acting on cones and obeying linear flow and jump rules. Finally, as a last contribution of this paper, we will show how to design V_p in item (V_p) to augment linear continuous-time control systems with hybrid loops that reduce the overshoot of a scalar plant output. In comparison to previous work, the aim of this paper is to design hybrid strategies to guarantee some asymptotic stability property by enforcing that suitable Lyapunov-like functions are not increasing along the hybrid solutions.

The arising hybrid closed loop resembles the so-called impulsive systems, considered e.g. in Haddad, Chellaboina, and Kablar (2001). However the objectives of Haddad et al. (2001) and of the present paper are different. Indeed an inverse optimal control involving a hybrid nonlinear-non-quadratic performance functional is developed in Haddad et al. (2001), whereas here we provide a design method of a hybrid loop (namely the jump map and the jump/flow sets) to ensure asymptotic stability and non-increase of suitable scalar functions. Our results are also linked to the event-triggered control literature (see Anta & Tabuada, 2010) for stability analysis of networked control systems, where it is necessary to reduce the number of times when the state is measured by the controller and the actuators are updated. The most important difference between the results mentioned above and our contribution is that in those works the resetting value for the state is uniquely associated to the transmission of a measurement sample, whereas in our results it depends on the Lyapunov-like functions that should not increase along solutions. Preliminary results in the direction of the work of this paper have been presented, without proofs, in Prieur, Tarbouriech, and Zaccarian (2010, 2011). Our preliminary work also contains additional examples, not reported here due to space constraints. The present paper provides an improved discussion of the preliminary results, together with their proofs.

2. Main results: nonlinear case

2.1. Constructing V_p from V

In this section we consider the closed-loop nonlinear system (3) and a function V of the closed-loop state to address item (V) of Section 1. To this aim, we make the following assumption on the function V.

Assumption 1. The function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is continuously differentiable such that there exists a continuous differentiable function $\phi : \mathbb{R}^{n_p} \to \mathbb{R}^{n_c}$ such that

$$\phi(x_p) \in \operatorname{argmin}_{x_c \in \mathbb{R}^{n_c}} V(x_p, x_c). \tag{6}$$

Moreover, there exists a class $\mathcal{K}$ function α such that, for all x_p in $\mathbb{R}^{n_p}$, $x_p \neq 0$,

$$\langle \nabla_p V(x_p, \phi(x_p)), f_p(x_p, \phi(x_p)) \rangle < -\alpha(V(x_p, \phi(x_p))) \tag{7}$$

where $\nabla_p V$ denotes the gradient of V with respect to its first argument.

Remark 1. In Assumption 1 we do not impose that (3) is globally asymptotically stable, because (7) requires the function V to be decreasing only in the subset of the state space defined by $(x_p, x_c) = (x_p, \phi(x_p))$. Nevertheless, if system (3) is globally asymptotically stable, then there exist a function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ and a class $\mathcal{K}$ function α such that $\langle \nabla V(x), f(x) \rangle < -\alpha(V(x))$ for all $x \neq 0$, which implies (7). Moreover note that in Assumption 1, it is not required that $\operatorname{argmin}_{x_c \in \mathbb{R}} V(x_p, x_c)$ is a single valued map, but only that a continuous differentiable selection of this map does exist. For example, with $V(x_p, x_c) = x_p^4 + x_c^4 - x_p^2 x_c^2$, we have $\operatorname{argmin}_{x_c \in \mathbb{R}} V(x_p, x_c) = \{x_p, -x_p\}$ which is not a singleton even though Assumption 1 can be satisfied, e.g., with $\phi(x_p) = x_p$. □

A natural way to stabilize the closed-loop system (3) is to flow when one (or both) of V and V_p is strictly decreasing and to reset the x_c-component of the state to the value $\phi(x_p)$ (where strict decrease is guaranteed by (7)) when the function is not decreasing. This leads to the following hybrid system[3]

$$\begin{aligned} &\dot{x} = f(x) \quad \text{if } x \in \hat{F}, \\ &(x_p^+, x_c^+) = (x_p, \phi(x_p)) \quad \text{if } x \in \hat{J}, \end{aligned} \tag{8}$$

where $\hat{F} \subset \mathbb{R}^n$ and $\hat{J} \subset \mathbb{R}^n$ are suitable closed subsets of the state space such that $\hat{F} \bigcup \hat{J} = \mathbb{R}^n$. In particular, $\hat{F}$ and $\hat{J}$ are defined by suitably combining the following two pairs of sets arising, respectively, from the knowledge of V and V_p:

$$\begin{aligned} F &= \{x \in \mathbb{R}^n,\ \langle \nabla V(x), f(x) \rangle \leq -\bar{\alpha}(V(x))\} \\ J &= \{x \in \mathbb{R}^n,\ \langle \nabla V(x), f(x) \rangle \geq -\bar{\alpha}(V(x))\} \end{aligned} \tag{9}$$

$$\begin{aligned} \bar{F} &= \{x \in \mathbb{R}^n,\ \langle \nabla V_p(x_p), f_p(x_p, x_c) \rangle \leq -\bar{\alpha}(V_p(x_p))\} \\ \bar{J} &= \{x \in \mathbb{R}^n,\ \langle \nabla V_p(x_p), f_p(x_p, x_c) \rangle \geq -\bar{\alpha}(V_p(x_p))\} \end{aligned} \tag{10}$$

where $\bar{\alpha}$ is any class $\mathcal{K}$ function such that $\bar{\alpha}(s) \leq \alpha(s)$ for all $s \geq 0$ (this will be denoted next by the shortcut notation $\bar{\alpha} \leq \alpha$). We state next our first main result whose proof is reported in Section 4.

[2] Here, to keep the discussion simple, it is assumed that V is continuously differentiable and that there exists $\phi(x_p) \in \operatorname{argmin}_{x_c \in \mathbb{R}^{n_c}} V(x_p, x_c)$, which implies (4).

[3] For an introduction of the hybrid systems framework used in this paper, see, e.g., the survey Goebel, Sanfelice, and Teel (2009) or the brief overview in Nešić, Teel, and Zaccarian (2011, Section 2).