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Synchrosqueezing-based time-frequency analysis of multivariate data

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ABSTRACT

The modulated oscillation model provides physically meaningful representations of time-varying harmonic processes, and has been instrumental in the development of modern time-frequency algorithms, such as the synchrosqueezing transform. We here extend this concept to multivariate signals, in order to identify oscillations common to multiple data channels. This is achieved by introducing a multivariate extension of the synchrosqueezing transform, and using the concept of joint instantaneous frequency multivariate data. For rigor, an error bound which assesses the accuracy of the multivariate instantaneous frequency estimate is also provided. Simulations on both synthetic and real world data illustrate the advantages of the proposed algorithm.

1. Introduction

Numerous observations in science and engineering exhibit time-varying oscillatory behavior that is not possible to characterize adequately by conventional Fourier analysis. This limitation was first addressed by the modulated oscillation model [1], which characterizes time-varying signals as amplitude and frequency modulated oscillations, thereby capturing the changing oscillatory dynamics of the signal. The univariate modulated oscillation model has since become a standard in analyzing time-varying signals, in fields ranging from communication theory [2] to biomedical engineering [3].

The notion of the univariate modulated oscillation has been recently extended to both bivariate and trivariate time-varying signals [4–6]; in the bivariate case the modulated oscillations are modeled as tracing an ellipse with the joint instantaneous frequency capturing the combined frequencies of the individual channels. This elliptic (ellipsoid) characterization has found applications in oceanography [7], where the underlying physical processes are well modeled as particles in 2D and 3D spaces which trace elliptical trajectories. For an arbitrary number of channels, the modulated multivariate oscillation has been proposed in [8,9], whereby the underlying model assumes one common oscillation that best fits all of the individual channel oscillations. For instance, the work in [9] identifies modulated multivariate oscillations using the multivariate extension of the wavelet ridge algorithm, a local optimization technique that identifies local maxima with respect to scale parameter in the wavelet coefficients, with the objective of extracting the local oscillatory dynamics of the signal. It should be noted that interest in time-frequency analysis of multichannel data has also recently been growing with multivariate data driven algorithms [10,11] that directly exploit multichannel interdependencies.

A recent class of time-frequency techniques, referred to as reassignment methods [12–14], aim to improve the "readability" (localization) of time-frequency representations [15].

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The synchrosqueezing transform (SST) [16,17] belongs to this class, it is a post-processing technique based on the continuous wavelet transform that generates highly localized time-frequency representations of nonlinear and nonstationary signals. Synchrosqueezing provides a solution that mitigates the limitations of linear projection based time-frequency algorithms, such as the short-time Fourier transform (STFT) and continuous wavelet transforms (CWT). The synchrosqueezing transform reassigns the energies of these transforms, such that the resulting energies of coefficients are concentrated around the instantaneous frequency curves of the modulated oscillations. As such, synchrosqueezing is an alternative to the recently introduced empirical mode decomposition (EMD) algorithm [18]; it builds upon the EMD, by generating localized time-frequency representations while at the same time providing a well understood theoretical basis.

However, despite all those efforts, multivariate time-frequency algorithms are still at their infancy. This is in stark contrast with the developments in sensor technology which have made readily accessible multivariate data (3D inertial body sensor and 3D anemometers). To this end, we develop a multivariate time-frequency algorithm based on SST that generates a compact time-frequency representation of multichannel signals, based on the principles developed in [8,9,19].

The organization of this paper is as follows: Section 2 introduces the notion of modulated multivariate oscillations and joint instantaneous frequency. Section 3 describes the SST, Section 4 presents the proposed multivariate extension of the synchrosqueezing algorithm, and Section 5 verifies the algorithm through simulations.

2. Modulated multivariate oscillations

Signals containing single time varying amplitudes and frequencies are readily described by the modulated oscillation model

$$x(t) = a(t)\cos\phi(t) \tag{1}$$

where $a(t)$ and $\phi(t)$ are respectively the instantaneous amplitude and phase, and are termed the canonical pair [2]. The application of the Hilbert transform to the original signal, yields the analytic signal $x_+(t)$ in the form

$$x_+(t) = a(t)e^{i\phi(t)} = x(t) + i\mathcal{H}\{x(t)\} \tag{2}$$

where $\mathcal{H}\{\cdot\}$ is the Hilbert transform operator, and $i = \sqrt{-1}$. The analytic signal $x_+(t)$ is complex valued and admits a unique time-frequency representation for the signal $x(t)$, based on the derivative of the instantaneous phase, $\phi(t)$.

Recently, the concept of univariate modulated oscillation has been extended to the multivariate case, in order to model the joint oscillatory structure of a multichannel signal, using the well understood concepts of joint instantaneous frequency and bandwidth. Extending the representation in (2), for multichannel signal $\mathbf{x}(t)$, we can construct a vector at each time instant t, to give a multivariate analytic signal

$$\mathbf{x}_+(t) = \begin{bmatrix} a_1(t)e^{i\phi_1(t)} \\ a_2(t)e^{i\phi_2(t)} \\ \vdots \\ a_N(t)e^{i\phi_N(t)} \end{bmatrix} \tag{3}$$

where $a_n(t)$ and $\phi_n(t)$ represent the instantaneous amplitude and phase for each channel $n = 1, \ldots, N$. The work in [9] proposed the joint instantaneous frequency (power weighted average of the instantaneous frequencies of all the channels) of multivariate data in the form:

$$\omega_x(t) = \frac{\Im\left\{\mathbf{x}_+^H(t)\frac{d}{dt}\mathbf{x}_+(t)\right\}}{\|\mathbf{x}_+(t)\|^2} = \frac{\sum_{n=1}^N a_n^2(t)\phi_n'(t)}{\sum_{n=1}^N a_n^2(t)} \tag{4}$$

where the symbol "$\Im$" denotes the imaginary part of a complex signal and $\phi_n'(t)$ is the instantaneous frequency for each channel.

Remark 1. It should be noted that for single channel multicomponent signals, the weighted average instantaneous frequency [20] has the same form as in (4) and is consistent with the joint instantaneous frequency.

Both measures of instantaneous frequency overcome a fundamental problem that arises when estimating instantaneous frequency of multiple modulated oscillations, that is, power imbalances between the components lead to instantaneous frequency estimates that are outside the bounds of the individual instantaneous frequencies [21].

The joint instantaneous frequency captures the combined oscillatory dynamics of multivariate signals, while the joint instantaneous bandwidth $\upsilon_x(t)$ captures the deviations of the multivariate oscillations in each channel from the joint instantaneous frequency, and is given by

$$\upsilon_x(t) = \frac{\left\|\frac{d}{dt}\mathbf{x}_+(t) - i\omega_x(t)\mathbf{x}_+(t)\right\|}{\|\mathbf{x}_+(t)\|}. \tag{5}$$

Therefore, the joint instantaneous bandwidth represents the normalized error of the joint instantaneous frequency estimate with respect to the rate of change of the multivariate analytic signal $\mathbf{x}_+(t)$. Inserting (3) into (5) results in the expression for the squared instantaneous bandwidth:

$$\upsilon_x^2(t) = \frac{\sum_{n=1}^N (a_n'(t))^2 + \sum_{n=1}^N a_n^2(t)(\phi_n'(t) - \omega_x(t))^2}{\sum_{n=1}^N a_n^2(t)}. \tag{6}$$

Remark 2. Observe that the instantaneous bandwidth depends upon the rate of change of the instantaneous amplitudes for each channel, as well as the deviation of the instantaneous frequencies in each channel from the combined joint instantaneous frequency. Large deviations of the individual instantaneous frequencies from the joint instantaneous frequency result in a large instantaneous bandwidth, implying that the multivariate signal would not be well modeled as a multivariate modulated oscillation.

It has been shown in [6] that the global moments of the joint analytic spectrum can be expressed in terms of the joint instantaneous frequency and bandwidth. The first and second global moments are termed the joint mean frequency and the joint global second central moments (multivariate bandwidth squared). As a result, given the