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Survey paper

Stability analysis for stochastic hybrid systems: A survey[☆]

Andrew R. Teel[a,1], Anantharaman Subbaraman[a], Antonino Sferlazza[b]

[a] Department of Electrical and Computer Engineering, University of California, Santa Barbara, CA 93106-9560, United States

[b] Department of Energy, Information Engineering, and Mathematical Models, University of Palermo, Palermo, 90128, Italy

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ABSTRACT

This survey addresses stability analysis for stochastic hybrid systems (SHS), which are dynamical systems that combine continuous change and instantaneous change and that also include random effects. We re-emphasize the common features found in most of the models that have appeared in the literature, which include stochastic switched systems, Markov jump systems, impulsive stochastic systems, switching diffusions, stochastic impulsive systems driven by renewal processes, diffusions driven by Lévy processes, piecewise-deterministic Markov processes, general stochastic hybrid systems, and stochastic hybrid inclusions. Then we review many of the stability concepts that have been studied, including Lyapunov stability, Lagrange stability, asymptotic stability, and recurrence. Next, we detail Lyapunov-based sufficient conditions for these properties, and additional relaxations of Lyapunov conditions. Many other aspects of stability theory for SHS, like converse Lyapunov theorems and robustness theory, are not fully developed; hence, we also formulate some open problems to serve as a partial roadmap for the development of the underdeveloped pieces.

1. Overview

Stochastic hybrid systems (SHS) are dynamical systems that combine continuous change and instantaneous change and that include random effects. Some of the earliest references that study systems with these features include Bellman (1954), Bergen (1960), Bertram and Sarachik (1959), Rosenbloom (1955), Samuels (1959) and Sworder (1969). Several important subclasses of SHS have been studied extensively in the literature for the last several decades. These subclasses include *stochastic switched systems* (Chatterjee & Liberzon, 2004, 2006b; Dimarogonas & Kyriakopoulos, 2004; Feng, Tian, & Zhao, 2011; Feng & Zhang, 2006; Filipovic, 2009), *impulsive stochastic systems* (Wu, Han, & Meng, 2004), *Markov jump systems* (Chatterjee & Liberzon, 2006a, 2007; Mariton, 1990; Zhu, Yin, & Song, 2009), *hybrid switching diffusions* (Deng, Luo, & Mao, 2012; Ghosh, Arapostathis, & Marcus, 1991, 1993; Hanson, 2007; Hespanha, 2005; Khasminskii, Zhu, & Yin, 2007; Mao, 1999; Mao, Yin, & Yuan, 2007; Mao & Yuan, 2006; Pang, Deng, & Mao, 2008; Yin & Zhu, 2010; Yuan & Lygeros, 2005a,b; Yuan & Mao, 2003), *impulsive switching diffusions* (Yang, Li, & Chen, 2009), *stochastic impulsive systems driven by renewal processes* (Antunes, Hespanha, & Silvestre, 2010, 2012, 2013a,b; Hespanha & Teel, 2006), *diffusions driven by Lévy processes* (Applebaum, 2009; Applebaum & Siakalli, 2009; Bass, 2004; Fujiwara & Kunita, 1985), *impulsive stochastic systems with Markovian switching* (Hu, Shi, & Huang, 2006; Wu & Sun, 2006), *piecewise-deterministic Markov processes* (Costa, 1990; Costa & Dufour, 2008; Davis, 1984, 1993; Dufour & Costa, 1999; Hordijk & van der Duyn Schouten, 1984; Jacobsen, 2006; Yushkevich, 1983, 1986), *stochastic hybrid automata* (Bujorianu, 2004; Hu, Lygeros, & Sastry, 2000), *general stochastic hybrid systems* (Bujorianu, 2012; Bujorianu & Lygeros, 2006; Liu & Mu, 2006, 2008, 2009; Wu, Cui, Shi, & Karimi, 2013), and *stochastic hybrid inclusions* (Teel, 2013). In the most general models, instantaneous change is triggered both randomly in time and also possibly by the state reaching a certain region of the state space; moreover, the continuous evolution may have a diffusive component and the state values after instantaneous change may be determined via a probability distribution.

Some major applications for which SHS models have been usedin the literature (see also Cassandras & Lygeros, 2010) include financial systems (Applebaum, 2009, §5.6, David et al.,

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E-mail addresses: teel@ece.ucsb.edu (A.R. Teel), anantharaman@umail.ucsb.edu (A. Subbaraman), antonino.sferlazza@unipa.it (A. Sferlazza).

[1] Tel.: +1 805 893 3616; fax: +1 805 893 3262.

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2009, Hamilton, 1989, Ishijima & Uchida, 2011, Nunno, Meyer-Brandis, Øksendal, & Proske, 2006, Schaller & Norden, 1997), air traffic management systems (Hu, Prandini, & Sastry, 2005; Pola, Bujorianu, Lygeros, & Benedetto, 2003; Prandini & Hu, 2008, 2009; Prandini, Hu, Lygeros, & Sastry, 2000; Watkins & Lygeros, 2003), communication networks and networked control systems (Antunes et al., 2013a; Bohacek, Hespanha, Lee, & Obraczka, 2003; Donkers, Heemels, Van De Wouw, & Hetel, 2011; Hespanha, 2004, 2005, 2007; Hespanha, Bohacek, Obraczka, & Lee, 2001; Tabbara & Nesic, 2008), biological systems (Batt et al., 2005; De Jong et al., 2003; Ghosh & Tomlin, 2001; Hu, Wu, & Sastry, 2004; Kærn, Elston, Blake, & Collins, 2005; Lygeros et al., 2008; Rao, Wolf, & Arkin, 2002; Singh & Hespanha, 2010; Wilkinson, 2012), and power systems (Dhople, Chen, DeVille, & Dominguez-Garcia, 2013; Malhamé, 1990; Malhamé & Chong, 1983; Malhamé & Chong, 1985; Ugrinovskii & Pota, 2005; Wang & Crow, 2011). For financial systems, appreciation and volatility rates in financial markets may be subject to random, abrupt switches based on perceptions of investors and other unmodeled aspects of the economy. Air traffic management systems must contend with aircraft mode switching together with some diffusive, stochastic influences on aircraft dynamics. Flows in communication networks may be affected by random dropouts or congestion, and communication protocols may contain different modes for different operating conditions. Some biological concentration dynamics combine deterministic continuous evolution with stochastic birth and death events and promoter switching. Power systems can involve randomly-varying loads, electronic noise, and mode switching.

As these application areas suggest, a solid grasp of stability theory for SHS is useful for analysis or design of a wide range of systems. Of special interest to the control community are feedback systems that employ logic variables and randomness, and that perform well in the presence of discrete components, mechanical impacts, and random phenomena. This fact motivates this paper, which is a survey of stability analysis for SHS. In Sections 2–3, we review the main subclasses of SHS that have appeared in the literature while re-emphasizing, like in Pola et al. (2003), the common features found in most of these models. Section 4 addresses a variety of stability properties that have been considered in the SHS literature and Sections 5–7 summarize the basic known results about these properties. Typically, these results are sufficient conditions for stability that are expressed in terms of Lyapunov function candidates and bounds on the system's "infinitesimal generator" applied to these functions. Such results are summarized in Section 5 and connected to the SHS literature in Section 6. Relaxations of Lyapunov conditions are considered in Section 7.

SHS constitute an important generalization of non-stochastic hybrid dynamical systems, for which significant breakthroughs in stability theory have been carved out over the last decade (Branicky, 1998; DeCarlo, Branicky, Pettersson, & Lennartson, 2000; Liberzon, 2003; Michel, Hou, & Liu, 2008), including converse Lyapunov theorems, which establish the existence of smooth Lyapunov functions for asymptotic stable compact sets, and a variety of robustness properties (Goebel, Sanfelice, & Teel, 2012). In contrast, these types of results for SHS are not yet fully developed. Hence, in addition to surveying existing stability results, we pose several open problems in an attempt to provide a partial roadmap for the development of additional pieces that are needed to complete the stability theory puzzle for SHS. This is the nature of Section 8.

To keep the survey focused, we do not delve into SHS with delays, stochastic functional differential equations, or singular SHS, though we note that special types of such systems have been studied in the literature and some sufficient conditions for stability exist (Benjelloun & Boukas, 1998; Boukas, 2006a; Cao, Lam, & Hu, 2003; Huang & Mao, 2011; Ma & Boukas, 2009; Mao, 2002; Mao, Matasov, & Piunovskiy, 2000; Mao & Shaikhet, 2000; Peng & Zhang, 2010; Wang, Qiao, & Burnham, 2002; Xia, Boukas, Shi, & Zhang, 2009; Yang, Xu, & Xiang, 2006; Yuan & Mao, 2004; Yue, Fang, & Won, 2003; Yue & Han, 2005; Yue & Won, 2001). We also do not spend time on linear (jump Markov) systems and associated linear matrix inequalities for stability, though such results are extensive in the literature (Aberkane, 2011; Basak, Bisi, & Ghosh, 1996; Bolzern, Colaneri, & De Nicolao, 2010; Boukas, 2006b; Boukas & Shi, 1998; de Souza, 2006; Dragan & Morozan, 2002; El Ghaoui & Ait Rami, 1996; Fang & Loparo, 2002; Feng, Loparo, Ji, & Chizeck, 1992; Fragoso & Baczynski, 2002a,b; Fragoso & Costa, 2005; Gerencsér & Prokaj, 2010; Karan, Shi, & Kaya, 2006; Loparo & Fang, 2004; Mariton, 1988, 1990; Wu, Ho, & Li, 2010; Xiong, Lam, Gao, & Ho, 2005; Zhang & Boukas, 2009). Space constraints also limit our discussion of the SHS literature's exploration of almost sure exponential stability (Deng et al., 2012; Mao, 1999; Mao et al., 2007; Pang et al., 2008; Xiang, Wang, & Chen, 2011; Yuan & Lygeros, 2005a), asymptotic stability in distribution (Yuan & Mao, 2003), and input-to-state stability (Wu et al., 2013; Zhao, Feng, & Kang, 2012).

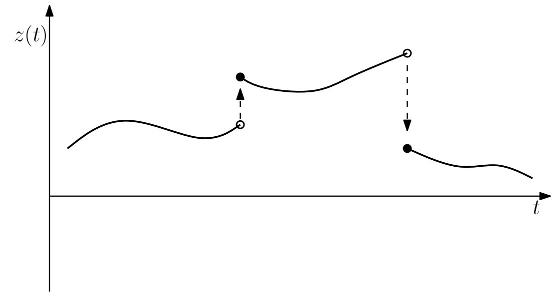

Fig. 1. A Càdlàg signal.

2. A unified framework

2.1. Solution candidates

Stochastic hybrid systems produce solutions defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ where Ω is the sample space, $\mathcal{F}$ is the event space, and $\mathbb{P}$ is the probability function defined on the event space. The symbol $\mathbb{E}$ is used for the associated expected values. We use $x \in \mathbb{R}^n$ to denote the state of a SHS. It may contain continuous-valued variables and discrete-valued variables, including logic variables, counters, and timers. For most of the SHS that we consider, solutions are measurable mappings $\mathbf{x} : \Omega \to \mathbb{D}([0,\infty), \mathbb{R}^n)$, where $\mathbb{D}([0,\infty), \mathbb{R}^n)$ denotes the space of Càdlàg functions from $[0,\infty)$ to $\mathbb{R}^n$. A function $\phi : [0,\infty) \to \mathbb{R}^n$ is Càdlàg if it is right continuous with left limits, that is, $\lim_{s\downarrow t}\phi(s) = \phi(t)$ for all $t \in [0,\infty)$ and $\phi(t^-) := \lim_{s\uparrow t}\phi(s)$ exists for all $t \in (0,\infty)$. See Fig. 1. A solution evaluated at random time $\mathbf{T} \geq 0$ is denoted $\mathbf{x}(\mathbf{T})$. Both $\mathbf{x}$ and $\mathbf{x}(\mathbf{T})$ are functions of $\omega \in \Omega$, though we rarely make the ω dependence explicit; the values of $\mathbf{x}$ belong to $\mathbb{D}([0,\infty), \mathbb{R}^n)$ while the values of $\mathbf{x}(\mathbf{T})$ belonging to $\mathbb{R}^n$. For a Borel set $C \subset \mathbb{R}^n$, we use $\mathfrak{B}(C)$ to denote the Borel σ-algebra on C; $\mathfrak{B}(C) = \cup_{A\in\mathfrak{B}(\mathbb{R}^n)} A \cap C$.

2.2. A common structure found in most models

Hybrid systems involve the combination of continuous change, called *flows*, and instantaneous change, called *jumps*. SHS allow the flows and the jumps to have random characteristics and also allow the timing of jumps to be random. SHS that have appeared in the literature include the following classes:

(1) switched and impulsive stochastic differential equations,
(2) systems with spontaneous jumps including:
   (a) Markov jump systems,
   (b) hybrid switching diffusions,