Signal Processing 102 (2014) 207–215

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Target localization in a multi-static passive radar system through convex optimization

Batu K. Chalise [a,1], Yimin D. Zhang [a,*], Moeness G. Amin [a], Braham Himed [b]

[a] Wireless Communications and Positioning Laboratory, Center for Advanced Communications, Villanova University, Villanova, PA 19085, USA
[b] Air Force Research Laboratory, AFRL/RYMD, Dayton, OH 45433, USA

ARTICLE INFO

Article history:
Received 14 September 2013
Received in revised form
14 January 2014
Accepted 26 February 2014
Available online 5 March 2014

Keywords:
Radar signal processing
Passive radar
Target localization
Convex optimization
Semi-definite relaxation

ABSTRACT

We propose efficient target localization methods for a passive radar system using time-of-arrival (TOA) information of the signals received from multiple illuminators, where the position of the receiver is subject to random errors. Since the maximum likelihood (ML) formulation of this target localization problem is a non-convex optimization problem, semi-definite relaxation (SDR)-based optimization methods in general do not provide satisfactory performance. As a result, approximated ML optimization problems are proposed and solved with SDR plus bisection methods. For the case without position error, it is shown that the relaxation guarantees a rank-one solution. The optimization problem for the case with position error involves only a relaxation of a scalar quadratic term. Simulation results show that the proposed algorithms outperform existing methods and provide root mean-square error performance very close to the Cramer–Rao lower bound.

1. Introduction

In recent years, multi-static passive radar (MPR) systems, which utilize multiple broadcast signals as sources of opportunity, have attracted significant interests due to their low cost, covertness, and availability of rich illuminator sources [1–4]. Compared to conventional active radar systems which typically operate in a monostatic mode and emit stronger signals with a wide signal bandwidth, MPR systems use broadcast signals which in general are very weak and have an extremely narrow bandwidth. These features make it difficult to exploit a MPR system for accurate target position estimation. In addition, MPR receivers may often be implemented on aerial or ground moving vehicles. In this case, the radar platform may only have inaccurate knowledge about its own instantaneous position. This uncertainty is caused by the accuracy limitation of the positioning system as well as multipath propagations.

Target localization is an important task that has received extensive attention in various applications, such as wireless communications, sensor networks, urban canyon, and through-the-wall radar systems [5–8]. Specifically, multi-lateration techniques utilize the range information observed at multiple positions, which are distributed over a region, to uniquely localize a target. Depending on the applications, range information can be obtained using time-of-arrival (TOA), time-delay-of-arrival (TDOA), and received signal strength indicator (RSSI). On the other hand, the observation positions may be achieved using fixed receivers, or synthesized using a single moving platform. In the latter case, the receiver positions are subject to inaccuracy.

In all these applications, maximum likelihood (ML) estimation is considered as a powerful method of estimating the targets' location, which in general is a non-convex optimization problem. When the measurement noise is sufficiently small, the ML estimation problem may be solved using linearized least squares (LLS) estimation methods [5,7]. The

* Corresponding author.
E-mail address: yimin.zhang@villanova.edu (Y.D. Zhang).
[1] Currently with Arraycomm LLC, San Jose, CA 95110, USA.

http://dx.doi.org/10.1016/j.sigpro.2014.02.023
0165-1684

key steps of the LLS estimation methods are linearizing the objective function using Taylor's series expansion at some initial guess of target position and updating it with the least squares (LS) solution in an iterative approach. Like in many iterative optimization techniques for non-convex problems, however, the accuracy of the LLS estimator highly depends on the initial guess of the target's location. This has motivated researchers to consider more efficient designs. One such approach is the semi-definite relaxation (SDR) technique [6,9–11], which converts a non-convex optimization problem into a convex one by relaxing certain rank constraints. It is worth noting that SDR-based approaches outperform computationally efficient two-step weighted least squares method proposed in [12], especially when the noise level is high and the sensor positions are not perfectly known.

The accuracy (or tightness) of SDR techniques, however, is problem specific, as shown in [13] for the TOA based optimization problems. For example, in optimization problems based on TDOA [9] and TOA [10], where an unknown time instant of the source's signal transmission is also optimized, SDR relaxations may not be tight and, thus, the penalty function approach is introduced. This is true also for robust designs where sensor positions are subject to certain random errors [9,10]. In this context, neglecting the second-order noise terms [12], the authors in [14] proposed to use an approximate ML function in the SDR-based source localization problem.

In this paper, we pursue approximated ML estimation approach in developing efficient target localization algorithms in a MPR system using TOA information of signals received from multiple illuminators and the target. As discussed above, the range resolution is poor because of the narrow signal bandwidth and weak signal levels, and the receiver position is subject to inaccurate knowledge of its own position. Therefore, an optimization problem is also formulated for the case where the receiver position is subject to estimation error. The underlying optimization problems are still non-convex, but can be reformulated as convex problems using SDR and solved in conjunction with the bisection method. When no position error is present, the SDR provides a rank-one solution. With position estimation error, the corresponding optimization problem involves only a relaxation of a quadratic scalar term.

The target localization problem and optimization technique described in this paper differs from the existing literature in a number of ways. In contrast to the optimization problems in [6,13], where the objective function is solely a function of monostatic range, the objective function in our case involves bistatic range, which makes accurate target position estimation much more challenging. Further, unlike [9,10], where SDR of several variables and a penalty function approach are employed, our approach involves SDR of only one variable, though in conjunction with the bisection approach. For these reasons, the proposed method outperforms methods in [9,10] and does not require refinement through local optimization. Moreover, although we employ ML approximation approach, the effect of the approximation renders our optimization problem to be different from [14] due to different system models. As a result, a new optimization method that solves relaxed SDR problem in conjunction with bisection method is proposed for both cases where receiver position is perfectly and imperfectly known.

The rest of the paper is organized as follows. The system model of the MPR system is described in Section 2, whereas the proposed optimization methods for target localization are presented in Section 3.2. The computational complexity of the proposed method is compared with the approach of [9] in Section 4. Numerical results are provided in Section 5 and conclusions are drawn in Section 6.

Notations: Upper (lower) bold face letters will be used for matrices (vectors); $(\cdot)^T$, $\mathbf{I}_n$, $\|\cdot\|$, $\mathrm{tr}(\cdot)$, $\mathbf{A}\succcurlyeq 0$, $\mathrm{diag}(\cdot)$ denote transpose, $n\times n$ identity matrix, Euclidean norm, matrix trace operator, positive semi-definiteness of $\mathbf{A}$, and diagonal matrix, respectively.

2. System model

We consider a standard MPR system with M illuminators of opportunity, which can be broadcast stations for digital-video broadcasting – terrestrial (DVB-T) [15] or base stations for global system for mobile communications (GSM) [16]. Because the deployment scenario of these broadcast and base stations is publicly available, their numbers and locations are considered to be precisely known. A narrow-band multi-frequency (NBMF) transmission is considered, where the illuminators use well separated carrier frequencies f_i, $i=1,\ldots,M$, and the bandwidths of their transmitted waveforms are much smaller than $f_i, \forall i$. The radar receiver observes the direct signals from all M illuminators and the reflected signal from a single target. The target is assumed to be stationary.[2] By virtue of NBMF transmission, separation of the different signals based on carrier frequencies (or equivalently illuminators) is feasible after demodulation and filtering [17]. It is worthwhile to note that this feature is in contrast to a multiple-input multiple-output (MIMO) radar system where transmitters use a same carrier frequency but coordinate to form orthogonal waveforms [18,19].

The TOA of the direct signal from the ith illuminator at the receiver, where $1\le i\le M$, is given by

$$\tau_{\mathrm{d},i}=\frac{1}{c}\|\mathbf{t}_i-\tilde{\mathbf{r}}\|, \tag{1}$$

where c is the speed of light, $\mathbf{t}_i$ and $\tilde{\mathbf{r}}$ are column vectors of length n that represent, respectively, the coordinates of the ith illuminator and the receiver. Depending on applications, n is 2 for a two-dimensional coordinate system and 3 for a three-dimension coordinate system. Note that $\{\mathbf{t}_i, \forall i\}$ are assumed to be stationary and precisely known. The TOA of the target reflected signal corresponding to the ith illuminator is given by

$$\tau_{\mathrm{b},i}=\frac{1}{c}\{\|\mathbf{t}_i-\mathbf{p}\|+\|\tilde{\mathbf{r}}-\mathbf{p}\|\}, \tag{2}$$

where $\mathbf{p}$ is the $n\times 1$ vector representing the location information of the target. Because the passive radar exploits non-cooperative illuminators, it does not know the exact

[2] Algorithms for multiple and moving targets will be reported elsewhere.