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A spline filter for multidimensional nonlinear state estimation

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ABSTRACT

The problem of nonlinear/non-Gaussian filtering has generated significant interest in the literature. The effectiveness of a nonlinear/non-Gaussian filtering algorithm depends on the accurate representation of the probability density function of the system state. The Extended Kalman Filter (EKF), the Approximate Grid Based methods (AGBM), and particle based filters have been developed to solve nonlinear/non-Gaussian problems. However, the EKF may not perform well with high nonlinearity due to its linear approximation; the AGBM often incurs extremely high storage/computational cost; particle filters provide weighted samples at discrete points in the state space instead of a continuous estimate of the probability density function of the system state, but most of them may suffer from the well-known degeneracy problem. In this paper, a comprehensive solution for nonlinear/non-Gaussian state estimation that can provide a continuous estimate of the probability density function of the system state is developed based on B-splines. The proposed spline filter is capable of modeling any arbitrary probability density function of the system state, and it is able to provide statistically the same estimation accuracy as the particle filter, but with only a few knots. It is also important to emphasize that, unlike most of the particle based algorithms (e.g., Sequential Important Sampling (SIS), Generic Particle Filter (GPF), and Sequential Monte Carlo (SMC) filter), the spline filter is free from degeneracy-like problems due to its continuous nature. In addition, by moving the knots dynamically, it ensures that the splines cover, and only cover, the regions where the probability of system state is significant so that the high efficiency of the spline filter is maintained. To make it applicable in common tracking applications, the spline filter is further extended to a multiple model one with the capability to handle systems with multiple maneuvering models. Besides theoretical derivations, simulation results are provided to verify the effectiveness of the proposed spline filter.

1. Introduction

The problem of nonlinear/non-Gaussian filtering has drawn significant interests in the field due to the inherent nonlinearity in most practical systems. For example, Ground Moving Target Indicator (GMTI) and Air Moving Target Indicator (AMTI) measurements include nonlinear components. Angle-only tracking [22], bistatic/multistatic tracking [9,13] are other examples with nonlinear measurement models. The nonlinearity in tracking problems

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may arise due to its presence in the state-to-measurement equation or in the evolution of the state itself. For example, due to road constraints and traffic patterns, the motion of ground targets may become highly nonlinear [35]. Air targets can exhibit high nonlinearity in their motion as well. The optimal nonlinear state estimator consists of the computation of the conditional (posterior) pdf of the multitarget state given all the measurements available up to the current time. Nonlinear transformations in state or measurements can also add to the complexity of the tracking problem by introducing non-Gaussian noise components. The effectiveness of nonlinear/non-Gaussian filtering depends on the accurate representation of the probability density function of the system state. The optimal state estimate solution of a linear Gaussian problem can be solved by using the Kalman filter (KF) [5]. Determining the optimal solution for the nonlinear non-Gaussian problem is much more challenging, but a suboptimal state estimate can be achieved by using approximation methods.

In the literature, various approaches have been proposed to find optimal/suboptimal solutions for the nonlinear filtering problem. For example suboptimal solutions can be found by linearizing the system equation and by approximating the non-linear posterior densities by a Gaussian density (e.g., the Extended Kalman Filter (EKF), Cubature Kalman Filter (CKF) [2,5]). Since the EKF approximates the true system state distribution by its mean and covariance, its performance degrades when the true distribution is non-Gaussian or highly nonlinear. The unscented transform was embedded into the EKF in [17,18], which results in the unscented Kalman filter (UKF). Although in some cases the UKF is shown to give better performance than the standard EKF [23], its ability to accurately solve the general non-Gaussian/nonlinear problem is limited.

In addition to the above point-based nonlinear filters, there are density-based ones as well [20,21]. The Gaussian Sum Filter (GSF) [3] represents the non-Gaussian posterior density by a Gaussian sum. This gives a continuous probability density function (pdf) from which a state estimate for a nonlinear non-Gaussian system can be found. The computational requirement of the GSF can be significantly greater than that of the EKF because the GSF is obtained as the convex combination of several Kalman filters. The performance of the GSF degrades with highly nonlinear system states [3]. The Approximate Grid-Based method (AGBM) filter uses numerical techniques [4], which simulate the state pdf by a set of grid points, but its limitation is the high storage/computational requirement. Besides, the discretization of the pdf by the AGBM using sampling leads to degeneracy [1]. Another issue with the AGBM is that the state space extent must be predefined and, therefore, cannot be partitioned nonuniformly to represent high probability regions with higher resolution, unless prior knowledge is available.

Alternatively, a point mass representation of probability densities can be found by using Particle Filters (PF) [1]. One of the basic particle filtering methods is the Sequential Important Sampling (SIS) algorithm and most of the particle filtering methods proposed in the literature can be directly derived from SIS algorithms. Some of the common PF methods are the Generic Particle Filter (GPF), the Regularized Particle Filter (RPF), the Sampling Importance Resampling (SIR) filter, and the Auxiliary Particle Filter (APF) [12]. The particle based approaches only provide a set of weighted samples at discrete points in the state space instead of a continuous estimate of the probability density function. A negative consequence of the discrete nature of particle filters is particle degeneracy. Although resampling can somewhat alleviate the degeneracy problem, it introduces the sample impoverishment problem [1]. The RPF can be used to solve degeneracy problem caused by sampling and resampling. However, the RPF has the disadvantage that the samples are no longer guaranteed to asymptotically approximate the posterior [25]. In practical scenarios with low measurement noise the RPF performs better than other particle based algorithms, but the performance advantage vanishes as the noise increases [1].

As compared to the discrete counterpart, the most substantial advantage of a filter providing continuous estimate of the probability density function is that continuous filter is free from degeneracy-like problems, because no sampling/resampling procedure is required. The Gaussian sum approximation in the GSF exemplifies a way to achieve this. In the literature, splines have been successfully used to represent complex (and arbitrarily continuous) curves and surfaces in computer science, graphics, aerospace, automobile industry, statistics and mathematics using a finite set of knots [19,28,34]. To find the continuous estimate of the system state, in [10], a spline approach is presented to solve the nonlinear estimation problem of phase demodulation. But, the system model is assumed to be an identical function with zero mean Gaussian noise, and thus the general state prediction step is not considered. An approach using monosplines to solve the nonlinear estimation problem is developed in [34], but with the assumption that the process and measurement noises are additive Gaussian, which may not be valid in general. In [27], a spline approach is developed to solve the one dimensional nonlinear/non-Gaussian filtering problem, and it has been extended to a multidimensional one in [28]. However, in [27,28], the system transition model is not explicitly modeled and a simulation based approach is used in the prediction step. After that, a spline interpolation is implemented to find the spline representation of the predicted state pdf. To explicitly model the system state transition, the use of the tensor products of splines to represent the system state transition model was provided in [19], but the corresponding analytical state prediction equation is not given and only a one-dimensional problem is considered.

In this paper, a spline filter (SF), which provides a comprehensive solution for general nonlinear non-Gaussian state estimation, is developed based on B-splines. Unlike the commonly used particle based Monte Carlo approaches that provide only weighted samples at discrete points in the state space, the new spline filter provides a continuous estimate of the probability density function of the state. The proposed spline filter can be extended to a multidimensional problems through the representation of multidimensional probability functions by tensor products of splines. In addition, this spline filter is able to provide statistically the same estimation