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Relaxed observer design of discrete-time T–S fuzzy systems via a novel multi-instant fuzzy observer

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ABSTRACT

This paper is concerned with the design of observer for discrete-time Takagi–Sugeno fuzzy systems. Under the framework of multi-instant matrix, a novel fuzzy observer and a new Lyapunov function, which are parameter-dependent on m-steps normalized fuzzy weighting functions, are proposed for conceiving less conservative observer design conditions. In particular, some existing fuzzy Lyapunov functions and fuzzy observers are special cases of the new Lyapunov function and fuzzy observer, respectively. Furthermore, the obtained fuzzy observer design conditions are further relaxed by fully considering the algebraic properties of m-steps normalized fuzzy weighting functions. Finally, a numerical example is given to illustrate the effectiveness of the proposed results.

1. Introduction

During the past several decades, state estimations for linear uncertain systems have attracted wide attention from scientists and engineers, essentially because the state variables in most practical control systems are not available [1–5]. So far, various methodologies have been developed for implementing the state estimation [6–9]. However, those aforementioned results of state estimations are only concerned with linear systems, but most of the real control systems belong to nonlinear systems and thus the above results fail to work in the process of state estimations such as nonlinear systems.

On the other hand, a well-known method to address the problems of control synthesis or state estimations for nonlinear systems is to represent the nonlinear system by using the Takagi–Sugeno (T–S) fuzzy model [10]. More importantly, it has been proved that the T–S fuzzy model can universally approximate any smooth nonlinear functions to any accuracy within a compact set by interpolation of local linear models with fuzzy membership functions [11]. Therefore, stability analysis and control synthesis of nonlinear systems could be investigated in view of the powerful control theory of linear systems, see [12–22] and references therein. Meanwhile, the problem of fuzzy state estimations has also been addressed widely by many researchers. For instance, several fuzzy H_∞ filters are developed for continuous-time T–S fuzzy dynamical systems [23,35,36], continuous-time T–S fuzzy dynamical systems with time delays [26,27,29–31,33,34], discrete-time T–S fuzzy dynamical systems with time delays [24,25,32,37], continuous-time T–S fuzzy dynamical systems with intermittent measurements [28], nonlinear Itô stochastic systems with application to sensor fault detection [38,39], multi-channel networked nonlinear systems with multiple packet dropouts [40], etc.

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However, it is worth noting that most of the existing state estimations are implemented by using the parallel distributed compensation (PDC) theory [41], and the obtained fuzzy observers/filters are only dependent on the current normalized fuzzy weighting functions. Therefore, the obtained fuzzy observer/filter conditions are rather conservative [42]. Recently, the authors in [42] have proposed a novel fuzzy observer for discrete-time T–S fuzzy systems, which is dependent on both the current normalized fuzzy weighting functions and one-step-past normalized fuzzy weighting functions, and thus a *better* result in the sense of solutions to the linear matrix inequality constraints problem has been obtained. Now, one will naturally raise a question: whether the conservatism could be further reduced if different fuzzy observer structures are adopted? In other words, the problem of seeking for more powerful fuzzy observer needs to be further investigated, which motivates us to carry out this work.

In this paper, an efficient multi-instant fuzzy observer which is parameter-dependent on m-steps ($m \geq 2$) normalized fuzzy weighting functions is proposed for the first time. The main contributions of this paper are summarized as follows: (I) one is reducing the conservatism of the existing fuzzy observer design by developing both a novel multi-instant fuzzy observer and a new Lyapunov function, which is parameter-dependent on m-steps ($m \geq 2$) normalized fuzzy weighting functions; (II) the other is extending the Polya Theorem into the higher-order homogeneous polynomials setting, thus the relations among different normalized fuzzy weighting functions are collected in a series of homogeneous matrix polynomials and the obtained fuzzy observer design conditions in (I) could be further relaxed. Since the information of the fuzzy systems could be fully considered, the relaxation quality could be significantly improved. Finally, an illustrative example is given to demonstrate the effectiveness of the proposed results.

Notations: The notation $P>0$ means that P is real symmetric and positive definite. $\mathbb{R}$ represents the set of real numbers, $\mathbb{Z}_+$ represents the set of positive integers, $\mathbb{N}$ denotes the natural numbers set $\{0,1,2,\ldots\}$, and $p!$ denotes the factorial, i.e., $p! = p(p-1)(p-2)\cdots(2)(1)$ for $p \in \mathbb{N}$ with $0! = 1$. Moreover, define the left-hand side of a relation as $\mathrm{Left}(\cdot)$, $\mathrm{He}(E)$ is defined as $\mathrm{He}(E) = E + E^T$.

2. Problem formulation and preliminaries

2.1. System descriptions and recent observer design conditions

Consider a class of discrete-time nonlinear system described by the following T–S fuzzy model:

Rule i: If $z_1(t)$ is F_1^i, and $z_2(t)$ is F_2^i,…, and $z_p(t)$ is F_p^i, then

$$\begin{cases} x(t+1) = A_i x(t) + B_i u(t), \quad i \in \{1,2,\ldots,r\}, \\ y(t) = C_i x(t). \end{cases}$$

where $t \in \mathbb{N}$, $x(t) \in \mathbb{R}^{n_1}$ is the state vector, $u(t) \in \mathbb{R}^{n_2}$ is the control input, $y(t) \in \mathbb{R}^{n_3}$ is the system output, and $z_j(t)$ is the measured premise variable.

By using product of inference, singleton fuzzifier, and center-average defuzzifier, the defuzzified output of the overall discrete-time T–S fuzzy model can be represented as follows:

$$\begin{cases} x(t+1) = \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(y)) \\ y(t) = \sum_{i=1}^{r} h_i(z(t)) C_i x(t). \end{cases} \tag{1}$$

where $h_i(z(t))$ denotes the i-th normalized fuzzy weighting function with the property of $h_i(z(t)) \geq 0$ and $\sum_{i=1}^r h_i(z(t)) = 1$ without loss of generality.

Recently, a new fuzzy observer has been designed in [42] for conceiving less conservative observer design conditions for (1):

$$\begin{cases} \hat{x}(t+1) = A_{z(t)}\hat{x}(t) + B_{z(t)}u(t) + G_{z(t-1)z(t)}^{-1} K_{z(t-1)z(t)}(y(t) - \hat{y}(t)), \\ \hat{y}(t) = C_{z(t)}\hat{x}(t). \end{cases} \tag{2}$$

where $\hat{x}(t)$ is the estimated state vector, $A_{z(t)} = \sum_{i=1}^r h_i(z(t))A_i$, $B_{z(t)} = \sum_{i=1}^r h_i(z(t))B_i$, $C_{z(t)} = \sum_{i=1}^r h_i(z(t))C_i$, $G_{z(t-1)z(t)} = \sum_{i=1}^r \sum_{j=1}^r h_i(z(t-1))h_j(z(t))G_{ij}$ and $K_{z(t-1)z(t)} = \sum_{i=1}^r \sum_{j=1}^r h_i(z(t-1))h_j(z(t))K_{ij}$. G_{ij} and K_{ij} are fuzzy observer matrices to be determined.

For consistency, the authors in [42] suppose that $z(-1) = z(0)$, and the estimation error system becomes

$$e(t+1) = (A_{z(t)} - G_{z(t-1)z(t)}^{-1} K_{z(t-1)z(t)} C_{z(t)}) e(t) \tag{3}$$

where $e(t) = x(t) - \hat{x}(t)$.

Then, by using an efficient Lyapunov function $V(e,z) = e^T(t)(\sum_{i=1}^r h_i(z(t-1))P_i)e(t)$, relaxed fuzzy observer design conditions are given as follows:

Proposition 1 (*Guerra et al.* [42]). *The estimation error* (3) *is globally asymptotically stable if there exist some matrices G_{ij}, P_i and K_{ij} for all $i,j \in \{1,2,\ldots,r\}$, such that the following linear matrix inequalities hold*:

$$\Upsilon_{ii}^k < 0, \quad i,k \in \{1,2,\ldots,r\}; \tag{4}$$

$$\frac{2}{r-1}\Upsilon_{ii}^k + \Upsilon_{ij}^k + \Upsilon_{ji}^k < 0, \quad i,j,k \in \{1,2,\ldots,r\}, i \neq j; \tag{5}$$

$$\Upsilon_{ii}^k = \begin{bmatrix} -P_k & * \\ G_{ik}A_i - K_{ik}C_i & -G_{ik} - G_{ik}^T + P_i \end{bmatrix},$$
$$\Upsilon_{ij}^k = \begin{bmatrix} -P_k & * \\ G_{jk}A_i - K_{jk}C_i & -G_{jk} - G_{jk}^T + P_j \end{bmatrix}.$$

2.2. Homogeneous polynomials and two useful lemmas

The following definitions are needed in the main proof, which are consistent with those in [19].

The set Δ_r is defined as $\Delta_r = \{\alpha \in \mathbb{R}^r; \sum_{i=1}^r \alpha_i = 1; \alpha \geq 0\}$. Define $\alpha_1^{k_1}\cdots\alpha_r^{k_r}, \alpha \in \Delta_r, k_i \in \mathbb{Z}_+, i = 1,2,\ldots,r$ are the monomials, $k = k_1 k_2 \cdots k_r$, and $P_k \in \mathbb{R}^{n \times n}, \forall k \in \mathcal{K}(g)$ are matrix-valued coefficients. Here, by definitions, $\mathcal{K}(g)$ is the set of r-tuples obtained as all possible combinations of nonnegative integers $k_i, i = 1,2,\ldots,r$, such that $k_1 + k_2 + \cdots + k_r = g$.