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Blind source separation of underdetermined mixtures of event-related sources

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ABSTRACT

This paper addresses the problem of blind source separation for underdetermined mixtures (i.e., more sources than sensors) of event-related sources that include quasi-periodic sources (e.g., electrocardiogram (ECG)), sources with synchronized trials (e.g., event-related potentials (ERP)), and amplitude-variant sources. The proposed method is based on two steps: (i) tensor decomposition for underdetermined source separation and (ii) signal extraction by Kalman filtering to recover the source dynamics. A tensor is constructed for each source by synchronizing on the "event" period of the corresponding signal and stacking different periods along the second dimension of the tensor. To cope with the interference from other sources that impede on the extraction of weak signals, two robust tensor decomposition methods are proposed and compared. Then, the state parameters used within a nonlinear dynamic model for the extraction of event-related sources from noisy mixtures are estimated from the loading matrices provided by the first step.

The influence of different parameters on the robustness to outliers of the proposed method is examined by numerical simulations. Applied to clinical electroencephalogram (EEG), ECG and magnetocardiogram (MCG), the proposed method exhibits a significantly higher performance in terms of expected signal shape than classical source separation methods such as πCA and FastICA.

1. Introduction

In this study, blind separation of underdetermined mixtures of event-related sources is addressed. An event-related source is characterized by typical patterns which are elicited after some events: such patterns may vary in amplitudes and/or in inter-event intervals (IEI). In this context, an event-related source is referred to as (i) quasi-periodic source (e.g., electrocardiogram (ECG)) in which IEI and amplitudes can only slightly change from a period to another; (ii) source with synchronized stimuli (e.g., event-related potentials (ERP)) in which a pattern is repeated with no assumption on IEI but with quasi-constant amplitudes; (iii) amplitude-variant source whose amplitude (even sign) can largely change from a period to another but with quasi-constant IEI (e.g., telecommunication); (iv) general source without any assumptions on amplitudes and IEI, which can thus largely vary from an event to another one (e.g., digital communications).

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In the recent years, a lot of attention has been paid to blind source separation (BSS) due to its wide-ranging applications in many areas [1] such as audio and speech processing [2], telecommunications [3], biomedical engineering [4], and hyperspectral imaging [5]. Assuming an M-dimensional observation vector, $\mathbf{y}(k)$, this problem is mathematically expressed as

$$\mathbf{y}(k) = \mathbf{A}\mathbf{x}(k) + \mathbf{b}(k), \tag{1}$$

where $\mathbf{x}(k)$ denotes the N-dimensional source vector, $\mathbf{b}(k)$ denotes the M-dimensional additive noise vector, and $\mathbf{A}$ is the $M \times N$ mixing matrix. The BSS framework aims at identifying the mixing matrix $\mathbf{A}$, or estimating the sources $\mathbf{x}(k)$, or both, from the observation $\mathbf{y}(k)$. Unlike the determined or overdetermined cases, when the number of sources is equal to or exceeds the number of mixtures ($N > M$), i.e., in the underdetermined case, the estimation of the mixing matrix $\mathbf{A}$ does not permit to directly recover the original sources. In fact, the mixing matrix does not admit a left inverse in that case, which makes it more difficult to recover the sources even if it is known and full rank [1,6]. It is then necessary to rely on a prior on the sources.

Sparsity of the sources in a transformed domain is a possible prior to address underdetermined BSS [7]. Indeed, most of the proposed methods in the literature of underdetermined BSS are based on the sparsity of sources in a domain (e.g., the frequency domain [8] or the time–frequency domain [9]). In this case, even if several sources are active at the same time so that the mixture is locally overdetermined, the mixing matrix can usually be estimated by clustering methods. However, this kind of search usually requires massive computations that limit the applicability of these methods to a smaller number of observation channels and sources [10].

Separation of underdetermined sources consists of two steps: estimation of the mixing matrix and extraction of the sources. Many algebraic and geometric (clustering) methods have been developed for the first step. They employ various decompositions of different data structures such as cumulant, correlation and cross-correlation matrices or tensors [1,10]. Then, a second step is required for recovering the original sources.

Higher-order tensors have gained increasing importance as they can be used to represent higher order cumulants that are exploited in independent component analysis (ICA) [11] and have been used successfully in BSS [12]. In addition, they are natural representations of multidimensional (higher than 2) data than matrices in many practical applications (e.g., in chemistry, biomedical engineering, and wireless communications). A fundamental challenge in these applications is to find informative and sparse representations of tensors, i.e., tensor decompositions. Tensor decompositions take into account information about different variables of the data, such as, for example, spatial, temporal and spectral information, and may provide links among the various extracted factors or latent variables with physical or physiological meaning and interpretation [13].

There are many applications in which the sources are known to be event-related. These properties are observed in digital communication, speech and some physiological signals such as electrocardiograms. The behavior of second- and fourth-order BSS algorithms in a cyclostationary context has been studied in [14]. In a recent study [10], an underdetermined separation method has been developed, which is suitable for separation of signals that are piecewise stationary, having time-varying variances. These algorithms which exploit the cyclostationarity property resort to statistical tools. In [15], a parallel deflation procedure based on a deterministic tensor decomposition has been proposed to address the problem of underdetermined BSS in the cyclostationary context. The basic approach consists in constructing a tensor by synchronizing on the symbol rate of a certain source, and decomposing the tensor using the Canonical Polyadic (CP) decomposition [16] to extract the characteristics of the source.

In this paper, the method described in [15] has been adapted for the estimation of the mixing matrix, temporal patterns, and amplitudes of event-related sources. The method described in [15] fails to extract a source which has very little power compared to the other sources because the latter act as interferers with high amplitudes that can be considered as outliers and impede on the accurate tensor decomposition. To overcome this problem, we propose to apply a robust tensor decomposition. In the literature, one can find several methods that have been developed to this end [17,18]. In general, these techniques are based on a modification of the classical quadratic cost function that is optimized during the tensor decomposition. For example, the authors of [18] suggested to minimize the mean absolute error, which reduces the impact of outliers in the data, but does not prevent them from influencing the results since high outliers still lead to high errors. It is also possible to introduce weights that account for different uncertainties of the tensor elements (see, e.g., [17]). In this paper, we present two robust CP decomposition methods. The first one, which we subsequently refer to as Gaussian CP (GCP) decomposition, goes a step further compared to the approach taken in [18] and optimizes a cost function that limits the maximal error to 1. The second method exploits the particular structure of the data to compute weights that discriminate outliers and employs a weighted CP (WCP) decomposition.

As the second step of the separation of underdetermined sources, a nonlinear state-space model has been developed for extracting N quasi-periodic sources (or components) from M observations. This model is used within a Kalman filtering framework, whose mixing matrix and state parameters are obtained from the loading matrices of the tensor decomposition.

The robustness of the proposed tensor decomposition methods to different parameters such as the initialization, the amount of outliers, the variability of the amplitudes, and synchronization errors is analyzed by means of numerical simulations. Furthermore, the proposed method is applied to biomedical data including electroencephalogram (EEG), ECG and magnetocardiogram (MCG) to extract desired sources. In the past, tensor-based techniques using classical CP decomposition have already been applied to space-time-realization EEG data (see [19–21]). However,