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A variable step-size sign algorithm for channel estimation

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ABSTRACT

This paper proposes a new variable step-size sign algorithm (VSSA) for unknown channel estimation or system identification, and applies this algorithm to an environment containing two-component Gaussian mixture observation noise. The step size is adjusted using the gradient-based weighted average of the sign algorithm. The proposed scheme exhibits a fast convergence rate and low misadjustment error, and provides robustness in environments with heavy-tailed impulsive interference.

1. Introduction

In recent years, the variable step-size (VSS) techniques have been adopted in the least-mean-square (LMS) algorithm for improving the convergence rate [1–9]. A VSS technique was proposed in [4] by applying the squared instantaneous error to control the step size. A variable step-size LMS (VSLMS) algorithm using the weighted average of the gradient vector was proposed in [5] and a variable step size normalized version (VSSNLMS) was proposed in [6]. A modified version of [4] using the noise resilient variable step size was presented in [7]. A quotient form LMS algorithm of filtered version of the quadratic error for system identification application was proposed in [8]. The LMS algorithm, which is applied to the sparse channel estimation, using an l_1-norm penalty to the cost function was proposed in [9]. The channel estimation is done by an adaptive filter, the weight vector of which is $\mathbf{w}_i=[w_{0,i},\ldots,w_{N-1,i}]^T$ with a tap length of N, and is updated based on the error e_i, which is given by

$$e_i=d_i-\mathbf{w}_i^T\mathbf{x}_i \tag{1}$$

and

$$d_i=y_i+n_i=\mathbf{w}_{\text{opt}}^T\mathbf{x}_i+n_i, \tag{2}$$

where $(\cdot)^T$, d_i, $\mathbf{x}_i$, y_i, n_i, and $\mathbf{w}_{\text{opt}}$ denote the vector transpose operator, the desired signal, the input signal vector $\mathbf{x}_i=[x_i,\ldots,x_{i-N+1}]^T$, the output of the unknown system, the system noise, and the optimal Wiener weight, respectively, at time index i. The algorithm for updating the weight of the LMS adaptive filter with a fixed step size μ is given as $\mathbf{w}_{i+1}=\mathbf{w}_i+\mu e_i\mathbf{x}_i$, where $e_i\mathbf{x}_i$ is the gradient vector. This is because the cost function using $(1/2)e_i^2$ is minimized according to the weights. The mathematical formulas used in these VSLMS algorithms to update the step size μ_i are summarized in Table 1. A common problem in these algorithms is that their convergence performance can be degraded by the presence of heavy-tailed impulsive interference. Because the energy of the instantaneous error is used as the cost function of the LMS algorithm [1–9] and the error signal is sensitive to impulsive noise, this will make these LMS-type algorithms prone to considerable degradation in several practical applications. Furthermore, because the error signal is used as an

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Table 1
Summary and complexity of the step-size updates of some existing VSLMS algorithms.

Algorithm	Update equations of the step size	The number of *mults* (*adds*)
VSS [4]	$\mu_i = \alpha\mu_{i-1} + \gamma e_i^2$	$2N+4$ $(2N+1)$
VSLMS [5]	$\begin{cases} \hat{\mathbf{p}}_i = \beta\hat{\mathbf{p}}_{i-1} + e_{i-1}\mathbf{x}_{i-1} \\ \mu_i = \mu_{i-1} + \gamma e_i \mathbf{x}_i^T \hat{\mathbf{p}}_i \end{cases}$	$5N+3$ $(4N)$
VSSNLMS [6]	$\begin{cases} \hat{\mathbf{p}}_i = \beta\hat{\mathbf{p}}_{i-1} + (1-\beta)\frac{\mathbf{x}_i}{\lVert\mathbf{x}_i\rVert^2} e_i \\ \mu_i = \mu_s \lVert\hat{\mathbf{p}}_i\rVert^2 \Big/ \lVert\mathbf{x}_i\rVert^2 \left(\frac{\sigma_n^2}{N\sigma_x^2} + \lVert\hat{\mathbf{p}}_i\rVert^2\right) \end{cases}$	$6N+6$ $(5N-1)$
Proposed	$\begin{cases} \hat{\mathbf{p}}_i = \beta\hat{\mathbf{p}}_{i-1} + (1-\beta)\mathrm{sgn}(e_i)\mathbf{x}_i \\ \mu_i = \alpha\mu_{i-1} + \gamma_s \lVert\hat{\mathbf{p}}_i\rVert^2 \end{cases}$	$5N+2$ $(4N)$

Note: the parameters represented by the same symbols in different algorithms are not necessarily related. The complexities of various algorithms include computation of the filter output and updates of the tap weights and step-size parameters (*mults* and *adds* denote the multiplications and additions, respectively).

Table 2
Summary and complexity of the step-size updates of some existing variable step-size sign algorithms.

Algorithm	Update equations of the step size	The number of *mults* (*adds*)
DSA [13]	$\begin{cases} r(e_i) = \begin{cases} \mathrm{sgn}(e_i), \lvert e_i\rvert \le \tau \\ L\,\mathrm{sgn}(e_i), \lvert e_i\rvert > \tau \end{cases} \\ \mu_i = \mu r(e_i) \end{cases}$	$2N+1$ $(2N)$
NRMN [14]	$\begin{cases} \lambda_i = 2\mathrm{erfc}[\lvert d_i\rvert/\hat{\sigma}_{d,i}] \\ \hat{\sigma}_{d,i} = \sqrt{\frac{1}{N-K_w-1}\mathbf{o}_i^T\mathbf{T}\mathbf{o}_i} \\ \mu_i = \frac{2A}{[2\lambda_i + [1-\lambda_i]\sqrt{2/\pi}(\sigma_b^2+\sigma_\eta^2)^{-1/2}]N\sigma_x^2} \end{cases}$	Greater than $3N-K_w+4$ $(3N-K_w+2)$
APSA [15]	$\mu_i = \mu/\sqrt{\lVert\mathbf{x}_i\rVert^2}$	$3N$ $(3N-1)$
MVSS-APSA [16]	$\begin{cases} \beta_i = \lambda\beta_{i-1} + (1-\lambda)\lvert e_{i-1}\rvert \\ \mu_i = \alpha\mu_i + (1-\alpha)\min\left(\frac{\lvert\lvert e_{i-1}\rvert - \beta_i\rvert}{\sqrt{\lVert\mathbf{x}_{i-1}\rVert^2}}, \ \mu_{i-1}\right) \end{cases}$	$3N+4$ $(3N+2)$
Proposed	$\begin{cases} \hat{\mathbf{p}}_i = \beta\hat{\mathbf{p}}_{i-1} + (1-\beta)\,\mathrm{sgn}(e_i)\mathbf{x}_i \\ \mu_i = \alpha\mu_{i-1} + \gamma_s \lVert\hat{\mathbf{p}}_i\rVert^2 \end{cases}$	$5N+2$ $(4N)$

Note: the parameters of $\mathbf{T}$ and $\mathbf{o}_i$ in the NRMN algorithm [14] are set according to $\mathbf{T}=\mathrm{Diag}[1, \ldots, 1, 0, \ldots, 0]$ and $\mathbf{o}_i = O([d_i, \ldots, d_{i-N+1}]^T)$. The $\mathbf{o}_i$ contains the most recent samples of d_i, ordered from the smallest to the largest absolute value ($O(.)$ denotes this ordering).

estimate of the step size, gradient-based algorithms are also sensitive to impulsive noise.

The sign algorithm (SA) [1–3,10–17], is now receiving attention in the adaptive filtering area because of the simplicity of its implementation. This algorithm can perform efficiently in the presence of impulsive interference. SA is more suitable for this application than LMS because it has a lower computational requirement and is resistant to the presence of impulsive interference. Based on the advantages of SA, several studies have used adaptive algorithms to reduce the detrimental effects of impulse noise. A robust mixed norm (RMN) algorithm using the weighted averaging of the l_1 and l_2 norms of error was proposed in [11] and its normalized version (NRMN) was introduced in [14]. A dual sign algorithm (DSA) operates between two sign algorithms with a large step-size parameter for increasing the convergence speed and a small one for reducing the steady-state error [12,13]. An affine projection sign algorithm (APSA) [15] using an l_1-norm optimization criterion has been proposed without involving any matrix inversion to achieve robustness against impulsive noise. A modified variable step-size APSA (MVSS-APSA) was proposed in [16] in order to obtain a fast convergence rate and small misalignment error when compared to APSA. A similar MVSS-APSA method applied to a subband adaptive filter was proposed in [17]. In [18], a variable sign-sign Wilcoxon algorithm was developed for the system identification application and performs efficiently in the presence of impulsive noise. The mathematical formulas used in these sign algorithms for updating the step size are summarized in Table 2.

This paper proposes a new framework based on scaling in the conventional SA cost function, using a critical factor γ to $\gamma\lvert e_i\rvert$ $(\gamma>0)$; hence, its gradient vector is $\gamma\,\mathrm{sgn}(e_i)\mathbf{x}_i$ and weight update is $\mathbf{w}_{i+1} = \mathbf{w}_i + \gamma\,\mathrm{sgn}(e_i)\mathbf{x}_i$. Similar to the step size, the parameter γ determines the convergence time and level of misadjustment of the algorithm. When the convergence speed of the SA is enhanced using a large step size, the convergence performance exhibits a substantial chattering phenomenon. The loss of information in the sign error signals occurs because they provide only positive or negative polarities, similar to a switching mode with a substantial chattering phenomenon in a control effect. To overcome this disadvantage, γ can be treated as a variable instead of a fixed