Signal Processing 102 (2014) 58–63

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Fast communication

Optimum sensor placement for fully and partially controllable sensor networks: A unified approach

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A R T I C L E I N F O

Article history:
Received 5 September 2013
Received in revised form
24 February 2014
Accepted 7 March 2014
Available online 15 March 2014

Keywords:
Cramer–Rao lower bound
Minimization
Optimum sensor array
Source localization
Time of arrival

A B S T R A C T

This paper considers the problem of optimum sensor placement in 2D for source localization using time of arrival measurements. We adopt a compact expression of the Fisher information matrix and derive the condition for sensor placements that will minimize the trace of the Cramer–Rao lower bound matrix of a source location estimate. The proposed placement criterion and solution framework apply to both fully and partially controllable sensor networks.

1. Introduction

For several decades, source localization has found wide applications in many fields, such as radar, sonar, wireless communications and recently sensor network [1–3]. Extensive research efforts have been put on the design of various algorithms to obtain a source location estimate that approaches the Cramer–Rao lower bound (CRLB) accuracy as much as possible. On the contrary, relatively limited attentions are focused on the impact of the sensor array geometry on the achievable localization accuracy.

For angle of arrival (AOA) or bearing-only localization, characterization of the optimal sensor-target geometry has been derived using various scalar measures of the CRLB or the Fisher information matrix (FIM) [4–7]. In time difference of arrival (TDOA) localization, Yang and Scheuing [8] derived the necessary and sufficient conditions for the optimum sensor array geometry that minimizes the CRLB and presented different placement strategies [9] to achieve this purpose. Similarly, by analyzing the geometry in terms of the Cramer–Rao inequality, Bishop et al. stated explicit results in terms of the relative angular geometry of the sensors with respect to the source for TOA localization [7,10]. Zhou et al. [11] also reached the same results for placing landmarks in TOA wireless localization. All these optimum sensor placement investigations assume that the position of every sensor is changeable, a scenario that will be referred to as a fully controllable sensor network. In practice, most of the sensors are at fixed locations and only a few can vary their positions, resulting in a partially controllable sensor network. In such a case, the optimum placement techniques found in the literature are not applicable.

In the paper, we derive condition and propose solution for optimum sensor placement in 2D TOA localization that are applicable to both fully and partially controllable sensor networks. The criterion of optimum placement is the trace of the CRLB of a source location estimate [7,10]. By adopting a compact expression of the FIM introduced in [12], both the fully and partially controllable sensor networks can be studied under a common framework. Although the results for fully controlled networks are available in the literature, the optimum sensor placement for partially controllable networks is the first time to be considered to our knowledge.

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http://dx.doi.org/10.1016/j.sigpro.2014.03.005
0165-1684/

2. Problem formulation

We study the optimum sensor placement problem in 2D. The localization geometry has M sensors at positions $\mathbf{s}_i$, $i=1,2,\ldots,M$ and they are used to locate an emitting source at unknown position $\mathbf{u}$, where $\mathbf{s}_i$ and $\mathbf{u}$ are 2D column vectors of Cartesian coordinates. This is achieved by observing the TOAs from the source to the sensors. Since the product of TOA with the signal propagation speed equals distance, we shall use TOA and distance (range) interchangeably.

Let r_i^o be the true distance between the source and the sensor i, hence from geometric relationship

$$r_i^o = \|\mathbf{u}-\mathbf{s}_i\| \tag{1}$$

where $\|*\|$ represents the 2-norm. The TOA measurement between the sensor at $\mathbf{s}_i$ and the source is

$$r_i = r_i^o + n_i \tag{2}$$

where n_i is the noise. The measurement vector contains the M TOAs $\mathbf{r}=[r_1,r_2,\ldots,r_M]^T=\mathbf{r}^o+\mathbf{n}$. The noise $\mathbf{n}$ is assumed to be a Gaussian distributed random vector with zero-mean and known covariance matrix $\mathbf{Q}_r=\mathrm{diag}(\sigma_1^2,\sigma_2^2,\ldots,\sigma_M^2)$.

Among all M sensors, the first $M-N$ of them have been deployed at fixed locations. The last N are the new sensors to be added to improve the localization accuracy of the unknown source. The objective is to determine where to place the N sensors in order to achieve the best possible localization performance.

3. CRLB

The CRLB, which is the inverse of the Fisher Information Matrix (FIM), provides the lower bound on the covariance matrix of any unbiased estimator. We shall use the trace of the CRLB as the optimization criterion for the sensor placement problem.

Based on the Gaussian TOA model and the definition of r_i^o in (1), the FIM of $\mathbf{u}$ is

$$\mathrm{FIM}(\mathbf{u}) = \mathrm{CRLB}(\mathbf{u})^{-1} = [\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\ldots,\boldsymbol{\rho}_M]\mathbf{Q}_r^{-1}[\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\ldots,\boldsymbol{\rho}_M]^T. \tag{3}$$

$\boldsymbol{\rho}_i$ is defined as a unit vector pointing from the source to the sensor at $\mathbf{s}_i$, i.e.

$$\boldsymbol{\rho}_i = \frac{\mathbf{s}_i-\mathbf{u}}{r_i^o} = [\cos\,\varphi_i,\ \sin\,\varphi_i]^T. \tag{4}$$

4. Optimum placement of the sensors

We first introduce a compact description of the FIM proposed in [12] that was used to illustrate the increase in Fisher Information of a source location estimate when a new sensor node is added in non-collaborative localization. We shall extend the FIM formula in Theorem 2 of [12] from having one new sensor to multiple and derive the optimum sensor placement.

From (3), separating the information of the fixed and the new sensors gives

$$\mathrm{FIM}(\mathbf{u}) = \mathrm{FIM}(\mathbf{u})_f + \sum_{i=M-N+1}^{M} \lambda_i \boldsymbol{\rho}_i \boldsymbol{\rho}_i^T \tag{5}$$

where $\lambda_i=\sigma_i^{-2}$ and $\mathrm{FIM}(\mathbf{u})_f=\sum_{i=1}^{M-N}\lambda_i\boldsymbol{\rho}_i\boldsymbol{\rho}_i^T$. Note that $\mathrm{FIM}(\mathbf{u})_f$ is a 2×2 matrix and its eigen-decomposition can be written as [12]

$$\mathrm{FIM}(\mathbf{u})_f = \mathbf{U}_\theta \begin{bmatrix} \mu & 0 \\ 0 & \eta \end{bmatrix} \mathbf{U}_\theta^T \tag{6}$$

where μ and η are the two eigenvalues with $\mu \ge \eta$. The corresponding eigenvectors are the columns of

$$\mathbf{U}_\theta = \begin{bmatrix} \cos\,\theta & -\sin\,\theta \\ \sin\,\theta & \cos\,\theta \end{bmatrix}. \tag{7}$$

Note that $\mathbf{U}_\theta$ is actually a rotation matrix with angle $-\theta$. Indeed, μ and η correspond to the minor and major axes of the concentration ellipse and θ the rotation angle. For ease of illustration in the following, the right-hand side of (6) is denoted by $\mathbf{F}(\mu,\eta,\theta)$.

Taking inverse of $\mathrm{FIM}(\mathbf{u})_f$ in (6) and computing the trace yield the localization accuracy when using the first $M-N$ fixed sensors

$$\mathrm{tr}\left(\mathrm{CRLB}(\mathbf{u})_f\right) = \mathrm{tr}\left(\mathrm{FIM}(\mathbf{u})_f^{-1}\right) = \frac{1}{\mu}+\frac{1}{\eta} \tag{8}$$

where the properties $\mathbf{U}_\theta^{-1}=\mathbf{U}_\theta^T$ and $\mathrm{tr}(\mathbf{ABC})=\mathrm{tr}(\mathbf{BCA})$ have been used.

Similarly, the eigen-decomposition of the entire FIM with all M sensors can also be written as

$$\mathrm{FIM}(\mathbf{u}) = \mathbf{F}(\mu^N,\eta^N,\theta^N) = \mathbf{F}(\mu,\eta,\theta) + \sum_{i=M-N+1}^{M} \lambda_i \boldsymbol{\rho}_i \boldsymbol{\rho}_i^T \tag{9}$$

where by extending Theorem 2 of [12] from one to N additional sensors,

$$\theta^N = \theta + \frac{1}{2}\arctan\frac{b}{\mu-\eta+a} \tag{10a}$$

$$\mu^N = \frac{\mu+\eta+\sum_{i=M-N+1}^{M}\lambda_i}{2} + \frac{\Delta}{2} \tag{10b}$$

$$\eta^N = \frac{\mu+\eta+\sum_{i=M-N+1}^{M}\lambda_i}{2} - \frac{\Delta}{2} \tag{10c}$$

$$\Delta = \sqrt{(\mu-\eta+a)^2+b^2} \tag{10d}$$

$$a = \sum_{i=M-N+1}^{M} \lambda_i \cos\,2\varphi_i' \tag{10e}$$

$$b = \sum_{i=M-N+1}^{M} \lambda_i \sin\,2\varphi_i' \tag{10f}$$

with $\varphi_i'=\varphi_i-\theta$. As a result, the localization accuracy when using all M sensors is

$$\mathrm{tr}(\mathrm{CRLB}(\mathbf{u})) = \frac{1}{\mu^N}+\frac{1}{\eta^N} = 4\frac{\mu+\eta+\sum_{i=M-N+1}^{M}\lambda_i}{(\mu+\eta+\sum_{i=M-N+1}^{M}\lambda_i)^2-\Delta^2}. \tag{11}$$

Note that only φ_i and hence φ_i' in (11) are variables that can be changed and all the others, μ, η, θ and λ_i are constant. Indeed, φ_i determine the position of the new N sensors relative to the source as can be seen from (4). Hence the optimum sensor placement problem is solved by minimizing $\mathrm{tr}(\mathrm{CRLB}(\mathbf{u}))$ in (11) which is equivalent to the minimization