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Fast communication

Sparse signal recovery from one-bit quantized data: An iterative reweighted algorithm ☆

Jun Fang [a], Yanning Shen [a], Hongbin Li [b,*], Zhi Ren [c]

[a] National Key Laboratory of Science and Technology on Communications, University of Electronic Science and Technology of China, Chengdu 611731, China
[b] Department of Electrical and Computer Engineering, Stevens Institute of Technology, Hoboken, NJ 07030, USA
[c] School of Communication and Information Engineering, Chongqing University of Posts and Telecommunications, Chongqing, China

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ABSTRACT

This paper considers the problem of reconstructing sparse signals from one-bit quantized measurements. We employ a log-sum penalty function, also referred to as the Gaussian entropy, to encourage sparsity in the algorithm development. In addition, in the proposed method, the logistic function is introduced to quantify the consistency between the measured one-bit quantized data and the reconstructed signal. Since the logistic function has the tendency to increase the magnitudes of the solution, an explicit unit-norm constraint is no longer necessary to be included in our optimization formulation. An algorithm is developed by iteratively minimizing a convex surrogate function that bounds the original objective function. This leads to an iterative reweighted process that alternates between estimating the sparse signal and refining the weights of the surrogate function. Numerical results are provided to illustrate the effectiveness of the proposed algorithm.

1. Introduction

Conventional compressed sensing framework recovers a sparse signal $\boldsymbol{x} \in \mathbb{R}^n$ from only a few linear measurements:

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} \tag{1}$$

where $\boldsymbol{y} \in \mathbb{R}^m$ denotes the acquired measurements, $\boldsymbol{A} \in \mathbb{R}^{m \times n}$ is the sampling matrix, and $m \ll n$. Such a problem has been extensively studied and a variety of algorithms that provide consistent recovery performance guarantee were proposed, e.g. [1,2]. In practice, however, measurements have to be quantized before being further processed. Moreover, in distributed systems where data acquisition is limited by bandwidth and energy constraints, aggressive quantization strategies which compress real-valued measurements into one or only a few bits of information are preferred. This has inspired recent interest in studying compressed sensing based on quantized measurements. Specifically, in this paper, we are interested in an extreme case where each measurement is quantized into one bit of information

$$\boldsymbol{b} = \text{sign}(\boldsymbol{y}) = \text{sign}(\boldsymbol{A}\boldsymbol{x}) \tag{2}$$

where "sign" denotes an operator that performs the sign function element-wise on the vector, the sign function returns 1 for positive numbers and -1 otherwise. Clearly, in this case, only the sign of the measurement is retained while the information about the magnitude of the signal is lost. This makes an exact reconstruction of the sparse signal $\boldsymbol{x}$ impossible. Nevertheless, if we impose a unit-norm on the

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* Corresponding author. Tel.: +1 201 216 5604; fax: +1 201 216 8264.
E-mail addresses: JunFang@uestc.edu.cn (J. Fang), 201121260110@std.uestc.edu.cn (Y. Shen), Hongbin.Li@stevens.edu (H. Li), renzhi@cqupt.edu.cn (Z. Ren).

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sparse signal, it has been shown [3,4] that signals can be recovered with a bounded error from one bit quantized data. Besides, in many practical applications such as source localization, direction-of-arrival estimation, and chemical agent detection, it is the locations of the nonzero components of the sparse signal, other than the amplitudes of the signal components, that have significant physical meanings and are of our ultimate concern. Recent results [5] show that asymptotic reliable recovery of the support of sparse signals is possible even with only one-bit quantized data.

The problem of recovering a sparse or compressible signal from one-bit measurements was first introduced by Boufounos and Baraniuk in their work [6]. Following that, the reconstruction performance from one-bit measurements was more thoroughly studied [3–5,7,8] and a variety of one-bit compressed sensing algorithms such as binary iterative hard thresholding (BIHT) [3,9], matching sign pursuit (MSP) [10], l_1 minimization-based linear programming (LP) [4], and restricted-step shrinkage (RSS) [11] were proposed. Although achieving good reconstruction performance, these algorithms either require the knowledge of the sparsity level [3,10] or are l_1-type methods that often yield solutions that are not necessarily the sparsest [4,11]. In this paper, we study a new method that uses the log-sum penalty function for sparse signal recovery. The log-sum penalty function has the potential to be much more sparsity-encouraging than the l_1 norm. By resorting to a bound optimization approach, we develop an iterative reweighted algorithm that successively minimizes a sequence of convex surrogate functions. The proposed algorithm has the advantage that it does not need the cardinality of the support set, K, of the sparse signal. Moreover, numerical results show that the proposed algorithm outperforms existing methods in terms of both the mean squared error and the support recovery accuracy metrics.

2. Problem formulation

Since the only information we have about the original signal is the sign of the measurements, we hope that the reconstructed signal $\hat{\boldsymbol{x}}$ yields estimated measurements that are consistent with our knowledge, that is

$$\text{sign}(\boldsymbol{a}_i^T \hat{\boldsymbol{x}}) = b_i \quad \forall i \tag{3}$$

or in other words

$$b_i \boldsymbol{a}_i^T \hat{\boldsymbol{x}} \geq 0 \quad \forall i \tag{4}$$

where $\boldsymbol{a}_i$ denotes the transpose of the ith row of the sampling matrix $\boldsymbol{A}$, b_i is the ith element of the sign vector $\boldsymbol{b}$. This consistency can be enforced by hard constraints [4,11] or can be quantified by a well-defined metric which is meant to be maximized/minimized [3,10,12]. In this paper, we introduce the logistic function to quantify the consistency between the measurements and the estimates. The metric is defined as

$$\phi(\boldsymbol{x}) \triangleq \sum_{i=1}^{m} \log(\sigma(b_i \boldsymbol{a}_i^T \boldsymbol{x})) \tag{5}$$

where $\sigma(x) \triangleq 1/(1+\exp(-x))$ is the logistic function. The logistic function, with an 'S' shape, approaches one for positive x and zero for negative x. Hence it is a useful tool to measure the consistency between b_i and $\boldsymbol{a}_i^T \boldsymbol{x}$. Also, the logistic function, differentiable and log-concave, is more amiable for algorithm development than the indicator function adopted in [3,10,12]. Note that the logistic function, also referred to as the logistic regression model, has been widely used in statistics and machine learning to represent the posterior class probability [13].

Naturally our objective is to find $\boldsymbol{x}$ to maximize the consistency between the acquired data and the reconstructed measurements, i.e.

$$\max_{\boldsymbol{x}} \quad \phi(\boldsymbol{x}) = \sum_{i=1}^{m} \log(\sigma(b_i \boldsymbol{a}_i^T \boldsymbol{x})) \tag{6}$$

This optimization, however, does not necessarily lead to a sparse solution. To obtain sparse solutions, a sparsity-encouraging term needs to be incorporated to encourage sparsity of the signal coefficients. The most commonly used sparsity-encouraging penalty function is l_1 norm. An attractive property of the l_1 norm is its convexity, which makes the l_1-based minimization a well-behaved numerical problem. Despite its popularity, l_1 type methods suffer from the drawback that the global minimum does not necessarily coincide with the sparsest solution, particularly when only a few measurements are available for signal reconstruction [14,15]. In this paper, we consider the use of an alternative sparsity-encouraging penalty function for sparse signal recovery. This penalty function, referred to as the Gaussian entropy, is defined as

$$h_G(\boldsymbol{x}) = \sum_{i=1}^{n} \log(x_i^2 + \epsilon) \tag{7}$$

where x_i denotes the ith component of the vector $\boldsymbol{x}$, and $\epsilon > 0$ is a small parameter to ensure that the function is well-defined. Such a log-sum penalty function was first introduced in [16] for basis selection and later more extensively investigated in [15,17–20]. This penalty function behaves more like the l_0 norm than the l_1 norm [15,21]. It can be readily shown that each individual log term $\log(x_i^2 + \epsilon)$, when $\epsilon \to 0$, has infinite slope at $x_i = 0$, $\forall i$, which implies that a relatively large penalty is placed on small nonzero coefficients to drive them to zero. Using this penalty function, the problem of finding a sparse solution to maximize the consistency can be formulated as follows:

$$\begin{aligned}\hat{\boldsymbol{x}} &= \arg\min L(\boldsymbol{x}) \\ &= \arg\min_{\boldsymbol{x}} -\sum_{i=1}^{m} \log(\sigma(b_i \boldsymbol{x}^T \boldsymbol{a}_i)) + \lambda \sum_{i=1}^{n} \log(x_i^2 + \epsilon) \end{aligned} \tag{8}$$

where λ is a parameter controlling the trade-off between the quality of consistency and the degree of sparsity. Note that for most state-of-the-art one-bit compressed sensing algorithms (e.g. [4,10,11]), a unit-norm constraint has to be imposed on the solution, otherwise the algorithms yield a trivial all-zero solution. Nevertheless, such a unit-norm constraint is non-convex [4,11]. To deal with the unit-norm constraint, sophisticated optimization techniques [11] or alternative constraints [4] need to be used. For our formulation, such a unit-norm constraint is no longer necessary. This is because the logistic function that is used to measure the sign consistency has a tendency to increase the magnitudes of the solution: note that the logistic

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