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Brief paper

Finite-time consensus for multi-agent networks with unknown inherent nonlinear dynamics☆

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ABSTRACT

The objective of this paper is to analyze the finite-time convergence of a nonlinear but continuous consensus algorithm for multi-agent networks with unknown inherent nonlinear dynamics. Due to the existence of the unknown inherent nonlinear dynamics, the stability analysis and the finite-time convergence analysis are more challenging than those under the well-studied consensus algorithms for known linear systems. For this purpose, we propose a novel comparison based tool. By using this tool, it is shown that the proposed nonlinear consensus algorithm can guarantee finite-time convergence if the directed switching interaction graph has a directed spanning tree at each time interval. Specifically, the finite-time convergence is shown by comparing the closed-loop system under the proposed consensus algorithm with some well-designed closed-loop system whose stability properties are easier to obtain. Moreover, the stability and the finite-time convergence of the closed-loop system using the proposed consensus algorithm under a (general) directed switching interaction graph can even be guaranteed by the stability and the finite-time convergence of some well-designed nonlinear closed-loop system under some special directed switching interaction graph. This provides a stimulating example for the potential applications of the proposed comparison based tool in the stability analysis of linear/nonlinear closed-loop systems by making use of known results in linear/nonlinear systems.

1. Introduction

The past decade has witnessed an increasing research interest in the study of distributed cooperative control of multi-agent networks due to its potential applications in military and civilian sectors. The main objective is to design local controllers for a team of agents such that a desired group behavior can be achieved. One typical scenario is consensus, whose objective is to design distributed control algorithms such that a group of agents reach an agreement on some state of interest. Existing research on consensus includes a deterministic interaction setting (Olfati-Saber & Murray, 2004), a stochastic interaction setting (Hatano & Mesbahi, 2005), and a sampled-data setting (Gao & Wang, 2009).

One particular area of interest is finite-time consensus, meaning that consensus is achieved in finite time. Compared with asymptotic consensus, meaning that consensus is achieved asymptotically, finite-time consensus has numerous benefits, such as a disturbance rejection property and robustness against uncertainties. Existing research in this area mainly concerns with agents with single-integrator kinematics (Cortes, 2006; Hui, Haddad, & Bhat, 2008; Sundaram & Hadjicostis, 2007; Wang & Xiao, 2010; Xiao, Wang, Chen, & Gao, 2009). In Cortes (2006), a nonsmooth consensus algorithm is proposed and its finite-time convergence is proved under an undirected fixed/switching interaction graph. In Hui et al. (2008), a continuous nonlinear consensus algorithm is proposed to guarantee the finite-time convergence under an undirected fixed interaction graph. In Wang and Xiao (2010), the algorithm in Hui et al. (2008) is shown to guarantee finite-time consensus under an undirected switching interaction graph and a directed fixed interaction graph when each strongly connected component of the graph is detail-balanced. In Xiao et al. (2009), another continuous nonlinear consensus algorithm is proposed to

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guarantee the finite-time convergence under a directed fixed interaction graph. In Cortes (2006), Hui et al. (2008), Wang and Xiao (2010), and Xiao et al. (2009), finite-time consensus is solved in a continuous-time setting. In Sundaram and Hadjicostis (2007), finite-time consensus is studied in a discrete-time setting, where the final equilibrium state can be computed after a finite number of time-steps.

Another area of interest is consensus for systems with unknown inherent nonlinear dynamics. Until now, consensus is primarily studied for agents with known system dynamics, assuming that system dynamics are fully identifiable. This assumption lacks practicality as system dynamics are often partially identifiable due to internal and external environment as well as system complexity. A typical mathematical mode for this type of systems is shown in Lu and Chen (2005), Lu, Yu, and Chen (2004) and Nishikawa, Motter, Lai, and Hoppensteadt (2003), where the dynamics of each agent in complex networks can be described by the sum of a continuously differentiable function describing the unknown inherent dynamics associated with the agent and the coupling item identifying the connection between the agent and other agents (see Eq. (1) in Lu & Chen, 2005; Lu et al., 2004; Nishikawa et al., 2003). Existing research in this area includes Su, Chen, Wang, and Lin (2011), Yu, Chen, and Cao (2011) and Yu, Chen, Cao, and Kurths (2010). In Yu et al. (2010), consensus is studied for second-order systems with unknown Lipschitz nonlinear dynamics. Sufficient conditions are derived to guarantee consensus under a directed fixed interaction graph. In Su et al. (2011), consensus is studied for second-order systems with unknown inherent nonlinear dynamics when a state-dependent communication graph is considered in the presence of a virtual leader. In Yu et al. (2011), consensus is studied for first-order systems in the presence of unknown inherent nonlinear dynamics under a directed fixed interaction graph. In Su et al. (2011), Yu et al. (2010) and Yu et al. (2011), consensus algorithms were developed to solve asymptotic consensus rather than finite-time consensus.

In this paper, we address finite-time consensus problem for a team of agents with unknown inherent nonlinear dynamics under a directed switching interaction graph by expanding on the preliminary work reported in Cao and Ren (2012). The uniqueness of our work is the consideration of both practical system dynamics (i.e., agents with unknown inherent nonlinear dynamics) and improved closed-loop performance (i.e., finite-time convergence). As the associated closed-loop system is a nonlinear time-varying system with unknown terms, its stability analysis becomes challenging. Instead of analyzing the stability of the original system, we show that the stability of the original system is guaranteed by the stability of another one by means of comparison. Such a novel comparison based approach requires the fulfillment of a few conditions. Detailed analysis shows how this comparison idea is used and how each condition is verified. This comparison based approach has the potential to be used in other problems.

The rest of this paper is organized as follows. Section 2 reviews notations used in this paper, graph theory notions, and the problem to be studied. Section 3 first proposes a consensus algorithm and then shows the finite-time convergence by means of comparison. A simulation example is provided in Section 4 to validate the proposed consensus algorithm. Section 5 concludes the paper with a brief discussion on future research.

2. Preliminaries and problem statement

2.1. Notations

$\mathbb{R}$ denotes the set of real numbers. $\mathbf{0}_n \in \mathbb{R}^n$ and $\mathbf{1}_n \in \mathbb{R}^n$ denote, respectively, the $n \times 1$ all-zero and all-one column vector. $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix. $\|\cdot\|$ denotes the 2-norm of a vector. $\emptyset$ denotes the empty set. Define $\text{sig}(x)^\alpha \overset{\triangle}{=} \text{sgn}(x)|x|^\alpha$, where $\text{sgn}(\cdot)$ denotes the signum function. Note that $\text{sig}(x)^\alpha$ is continuous in x when $\alpha > 0$. Let $f : [0,\infty) \mapsto \mathbb{R}^n$ be a continuous function. The upper Dini derivative of $f(t)$ is given by $D^+f(t) = \limsup_{h\to 0+} \frac{1}{h}[f(t+h) - f(t)]$. A function $f(t,x) : \mathbb{R}\times\mathbb{R}^n \mapsto \mathbb{R}^n$ is *globally* $\chi_1 - \chi_2$ Lipschitz in x if there exist $C \in (0,\infty]$, $\chi_1 \in (0,1]$, and $\chi_2 \in (0,1]$ such that $\|f(t,y_1) - f(t,y_2)\| \le C(\|y_1-y_2\|^{\chi_1} + \|y_1-y_2\|^{\chi_2})$ for all $y_1,\ y_2 \in \mathbb{R}^n$.

2.2. Graph theory notions

For a team of n agents, the interaction among them can be modeled by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{W})$, where $\mathcal{V} = \{1, 2, \dots, n\}$ and $\mathcal{W} \subseteq \mathcal{V}^2$ represent, respectively, the agent set and the edge set. An edge denoted as (i,j) means that agent j can obtain information from agent i. That is, agent i is a neighbor of agent j. We use $\mathcal{N}_j$ to denote the neighbor set of agent j. A directed path is a sequence of edges of the form $(i_1,i_2), (i_2,i_3), \dots$, where $i_k \in \mathcal{V},\ k = 1, 2, \dots$. A directed graph has a directed spanning tree if there exists at least one agent that has directed paths to all other agents.

Two matrices can be used to represent an interaction graph: adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ with $a_{ij} > 0$ if $(j,i) \in \mathcal{W}$ and $a_{ij} = 0$ otherwise, and Laplacian matrix $\mathcal{L} = [\ell_{ij}] \in \mathbb{R}^{n\times n}$ with $\ell_{ii} = \sum_{j=1,j\ne i}^n a_{ij}$ and $\ell_{ij} = -a_{ij}, i \ne j$. Let $a_{ii} = 0,\ i = 1, \dots, n$, (*i.e.*, agent i is not a neighbor of itself). It can be verified that $\mathcal{L}$ has at least one eigenvalue equal to 0 with a corresponding right eigenvector $\mathbf{1}_n$.

2.3. Problem statement

As demonstrated in complex networks (see Eq. (1) in Lu & Chen, 2005; Lu et al., 2004; Nishikawa et al., 2003), agent dynamics are often described by

$$\dot r_i = \phi(t, r_i) + u_i, \quad i = 1, \dots, n, \tag{1}$$

where $r_i \in \mathbb{R}$ is the state of the ith agent, $\phi(t,r_i) \in \mathbb{R}$ is the unknown inherent nonlinear dynamics for the ith agent, and $u_i \in \mathbb{R}$ is the control input characterizing the coupling among these agents. In the consensus (equivalently, synchronization) of complex networks (Lu & Chen, 2005; Lu et al., 2004; Nishikawa et al., 2003), u_i is typically given by $u_i = -c\sum_{j=1}^n a_{ij}(r_i - r_j)$, where c is the coupling gain. When consensus is achieved, $u_i = 0$ and the nominal agent dynamics can be described as $\dot x = \phi(t,x)$. As $\phi(t,x)$ is nonzero in general, agents are not stationary even if their coupling terms are all zero. This reflects the complicated ways that complex networks evolve. Consensus is feasible when the unknown term $\phi(t,r_i)$ satisfies the following Lipschitz condition as

$$|\phi(t,r_i) - \phi(t,r_j)| \le \gamma|r_i - r_j|, \tag{2}$$

where γ is a known positive constant. In Yu et al. (2011), consensus is studied for agents with dynamics (1) when $\phi(t,r_i)$ satisfies a similar condition to (2). Sufficient conditions are derived such that consensus is achieved asymptotically under a directed switching interaction graph.

The objective of this paper is to design a control algorithm such that consensus is achieved in finite time for agents with dynamics (1) subject to (2). In other words, u_i is needed to guarantee that $|r_i(t) - r_j(t)| \to 0$ in finite time for all $i,j = 1, \dots, n$.

3. Control algorithm and stability analysis

The proposed nonlinear consensus algorithm has the form of

$$u_i = -\beta\sum_{j=1}^n a_{ij}(t)\text{sig}(r_i - r_j)^{\alpha(|r_i - r_j|)}, \tag{3}$$