Signal Processing 101 (2014) 266–277

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Blind equalization based on pdf fitting and convergence analysis

Souhaila Fki, Malek Messai, Abdeldjalil Aïssa-El-Bey, Thierry Chonavel

Institut Mines-Télécom, Télécom Bretagne, Université européenne de Bretagne, UMR CNRS 6285 Lab-STICC, Technopôle Brest Iroise, CS 83818 29238 Brest, France

ARTICLE INFO

Article history:
Received 5 August 2013
Received in revised form
8 January 2014
Accepted 11 February 2014
Available online 20 February 2014

Keywords:
Blind equalization
pdf
Parzen windowing
Performance analysis
ODE

ABSTRACT

In this paper, we address M-QAM blind equalization by fitting the probability density functions (pdf) of the equalizer output with the constellation symbols. We propose two new cost functions, based on kernel pdf approximation, which force the pdf at the equalizer output to match the known constellation pdf. The kernel bandwidth of a Parzen estimator is updated during iterations to improve the convergence speed and to decrease the residual error of the algorithms. Unlike related existing techniques, the new algorithms measure the distance error between observed and assumed pdfs for the real and imaginary parts of the equalizer output separately. The advantage of proceeding this way is that the distributions show less modes, which facilitates equalizer convergence, while as for multi-modulus methods phase recovery keeps being preserved. The proposed approaches outperform CMA and classical pdf fitting methods in terms of convergence speed and residual error. We also analyse the convergence properties of the most efficient proposed equalizer via the ordinary differential equation (ODE) method.

1. Introduction

In transmissions, multipath propagation introduces intersymbols interference (ISI) that can make it difficult to recover transmitted data. Thus, an equalizer can be used to reduce the ISI. Time domain equalization or alternatively frequency domain equalization can be considered. The latter is very interesting for broadband wireless communications. Indeed, for long channels it is computationally simpler than the corresponding time domain equalization with the same efficiency. However, in this paper, we consider multipath channels with length less than or equal to about 10 and time domain equalization is well suited in this case because then both approaches have approximately the same complexity and the same performance [1]. Moreover, in this paper, we are interested in continuous flow transmission rather than block transmission where frequency-domain equalization is more relevant.

Without knowledge of the channel, the first equalization methods rely on periodic transmission of training sequences that are known from the receiver. Then, adaptation of the equalizer coefficients is done by minimizing a cost function that measures some distance between the actual equalizer output and the desired reference signal. When the transmitter sends a training sequence, the equalizer taps can be easily adapted by using a stochastic optimization technique such as the Least Mean Squares (LMS) algorithm, the cost function of which minimizes the expectation of the squared error [2]. However, in many digital communication systems, the transmission of a bandwidth consuming training sequence is not suitable. In order to avoid training, blind equalization techniques have been developed to retrieve symbols transmitted through an unknown channel by only using received data and some knowledge upon the statistics of the original sequence. There exist many blind algorithms. Sato algorithm [3] was the first blind technique proposed. The Godard

E-mail address: souhaila.fki@telecom-bretagne.eu (S. Fki).

http://dx.doi.org/10.1016/j.sigpro.2014.02.009
0165-1684

algorithm [4] and the Constant Modulus Algorithm (CMA) [5], which is a particular case of Godard algorithm, are probably the most popular blind equalization techniques. However, they require a long data sequence to converge and show relatively high residual error. To overcome these limitations, several approaches have been proposed in the literature. For instance, we can mention the Normalized-CMA (NCMA) that accelerates convergence by estimating the optimal step size of the algorithm at each iteration [6]. The CMA with a gain stage, where the latter is inserted after the equalizer to control the behaviour of its output power for faster convergence, was proposed in [7]. The square contour algorithm minimizes dispersion of the equalizer output from a square for blind equalization of QAM modulations [8], while the regional multimodulus algorithm, also designed for QAM modulations, performs similar to the supervised normalized least-mean-squares algorithm [9]. The Modified Constant Modulus Algorithm (MCMA) also known as Multi-Modulus Algorithm (MMA) performs blind equalization and carrier phase recovery simultaneously [10], by measuring the errors of real and imaginary parts of the equalizer output separately. The minℓ_1-MMA and MGauss-MMA algorithms [11] outperform the MMA by combining the multi-modulus criterion with an alphabet-matching penalty term.

In the last decade, new blind equalization techniques, based on information theoretic criteria and pdf estimation of transmitted data, have been proposed. These criteria are optimized adaptively, in general by means of stochastic gradient techniques. Among these techniques, Renyi's entropy has been used as a cost function [12]. It involves pdf estimation with the Parzen window kernel method. This equalizer is very sensitive to noise and provides excellent results for some channels but fails to equalize some others. So, an alternative criterion based on forcing the pdf at the equalizer output to match the known constellation pdf has been proposed in [13]. As a cost function, it uses the Kullback–Leibler Divergence (KLD) between the pdfs. The Euclidean distance has also been proposed in [14]. It uses Parzen window with Gaussian kernels for pdf estimation. In [15], a technique based on fitting the pdf of the equalizer output at some relevant points that are determined by the modulus of the constellation symbols was proposed. It is known as sampled-pdf fitting. The authors of [15] also proposed in [16] the Stochastic blind equalization approach that uses the Quadratic Distance (SQD) between the pdf at the equalizer output and the known constellation pdf as a cost function. This method is designed for multilevel modulations and works at symbol rate. Many digital transmission systems with a high number of states use QAM modulations. As the multi-modulus approaches are well suited for such modulations, we propose to use these techniques to equalize QAM constellations. Therefore, in this paper, we propose a new family of blind algorithms based on the SQD fitting, that we call Multi-Modulus SQD-ℓp (MSQD-ℓp). Unlike the method in [15], MSQD-ℓp measures the distance error between observed and assumed pdfs for real and imaginary parts of the equalizer output separately. The advantage of proceeding this way is that involved distributions show less modes, leading thus to reduced complexity, while preserving phase recovery as for multi-modulus methods. In addition, we benefit from the fact that 1D pdfs can be accurately estimated with less data than 2D pdfs. In this paper, we are particularly interested in the case $p=1$ that leads to the MSQD-ℓ1 algorithm that involves the absolute values of the real and the imaginary parts of the equalizer output. Thus, the shape of equalized constellation modes is Gaussian which is in accordance with the statistical behavior of received data from a single path propagation, what the equalizer tries to achieve.

These techniques are designed for multilevel modulations, work at the symbol rate and admit a simple stochastic gradient-based implementation. For pdf estimation, we use the Parzen window. The proposed methods outperform CMA and classical pdf fitting approaches, in terms of convergence speed and residual error. As much as possible, it is interesting to analyze the convergence properties of blind equalizers to better understand their performance. In this paper, we focus on performance analysis of the MSQD-ℓ1. To this goal, we employ the Ordinary Differential Equation (ODE) method. Indeed, the ODE approach supplies a solid theoretical framework for such a task [17]. The exact convergence analysis of adaptive blind equalization algorithms is often difficult because they are derived from nonlinear criteria. Therefore, the convergence analysis of the MSQD-ℓ1 is conducted under some usual assumptions that are commonly met in the related literature. The contributions of this paper to the field of blind equalization include

1. A new family of blind equalization algorithms named MSQD-ℓp that converges faster than the CMA and the classical SQD pdf fitting [16] and achieves lower residual error.
2. Convergence and performance analysis of the most effective MSQD-ℓ1 algorithm based on the ODE method.

This paper is organized as follows. In Section 2, we present the blind equalization problem and the SQD pdf fitting method. In Section 3, we propose the new cost functions and their corresponding stochastic gradient expressions. The convergence and performance analysis of the MSQD-ℓ1 algorithm is developed in Section 4. Simulations are presented in Section 5 and conclusions of our work are given in Section 6.

2. Signal and equalizer model

2.1. Signal model

To transmit digital data, a sequence $\{\boldsymbol{s}(n)\}_{n\in\mathbb{Z}}$ of independent identically distributed (i.i.d) complex symbols belonging to a digital modulation constellation is sent through a channel of length L_h with impulse response $\boldsymbol{h}=[h_0, h_1, \ldots, h_{L_h-1}]^T$, where $(\cdot)^T$ denotes the transpose operator. Transmitted data are affected by multipath propagation, resulting in intersymbol interference (ISI) at the receiver side. Therefore, an equalizer is used to reduce this ISI. In our work, we are interested in blind equalization that only requires knowledge of the modulation used to send the data. The basic scheme of a blind equalization