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Fast communication

Non-uniform sampled cubic phase function

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ABSTRACT

Parameter estimation of polynomial phase signals (PPSs) based on the cubic phase function (CPF) and its extensions cannot be performed by using the fast Fourier transform (FT) algorithm. Therefore, in order to express the CPF by means of the FT, in this paper we propose a scheme for the CPF evaluation based on non-uniform sampling. Calculation complexity of the estimation procedure is significantly reduced, whereas the accuracy is the same or better compared to the original algorithm.

1. Introduction

The cubic phase function (CPF) has been proposed for the estimation of third order polynomial phase signals (PPSs) [1]. Several generalizations of the CPF to PPSs of higher orders exist in the literature, including the recently proposed hybrid CPF–high-order ambiguity function (CPF–HAF) [2,3]. However, since the CPF-based techniques cannot be evaluated using the fast Fourier transform (FFT), they are characterized by larger computational complexity than the FT-based techniques such as the HAF and product HAF (PHAF) [4,5]. With the requirement of $O(N^2)$ complex multiplications and additions, where N is the number of signal samples, the CPF can be unfeasible in real-time applications, for example, in radar and sonar applications [3].

A procedure for parameter estimation of higher order PPSs based on the non-uniform signal sampling has been proposed in [6,7]. By sampling the signal at non-equidistant time instants, the procedure lowers the non-linearity of the estimator function, which, in turn, improves its performance, signal to noise ratio (SNR) threshold and mean squared error (MSE). In this paper, we use a similar non-uniform sampling scheme to transform the CPF in a form suitable for the FFT algorithm. The modified CPF requires $O(N \log N)$ operations and has lower MSE than the standard CPF. It can be generalized for other CPF-based techniques. Here, we implement the modified CPF in the evaluation of the CPF–HAF.

The rest of paper is organized as follows. In Section 2, we present the signal model and overview some of the most popular PPS estimators. The proposed method is presented in Section 3. In this section, the performance study is given as well. Numerical examples supporting the theoretical analysis are given in Section 4 followed by the conclusions in Section 5.

2. CPF and related estimators

Consider the following signal model:

$$x(n) = s(n) + \nu(n) = A \exp\left(j \sum_{i=0}^{P} a_i (n\Delta)^i\right) + \nu(n), \quad n \in [-N/2, N/2], \tag{1}$$

where $\nu(n)$ is complex zero-mean white Gaussian noise with variance σ^2 and $s(n)$ is a P-th order PPS with the amplitude A and phase parameters a_i, $i = 0, \ldots, P$. The number of signal samples is $N+1$ and Δ is the sampling rate. Here, N is even positive integer and Δ satisfies the

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Nyquist–Shannon sampling theorem. Our interest is to estimate $\{A, a_0, \dots, a_P\}$ from $x(n)$.

The maximum likelihood (ML) estimation of parameters $\{A, a_0, \dots, a_P\}$ of higher order PPSs is computationally complex since it requires maximization of a P-dimensional function [8]. Therefore, the phase differentiation (PD)-based techniques are used to reduce the search space [4,5,9]. The PD is performed recursively by the auto-correlation function

$$\mathrm{PD}^{K}_{x(n)}[n;\tau_1,\tau_2,\dots,\tau_K]=\mathrm{PD}^{K-1}_{x(n)}[n+\tau_K;\tau_1,\tau_2,\dots,\tau_{K-1}]\times\{\mathrm{PD}^{K-1}_{x(n)}[n-\tau_K;\tau_1,\tau_2,\dots,\tau_{K-1}]\}^*,\quad \mathrm{PD}^{0}_{x(n)}[n]=x(n), \tag{2}$$

where K is the number of PDs, $\tau_1, \tau_2, \dots, \tau_K$ are the lag parameters and $\mathrm{PD}^{K}_{x(n)}[n;\tau_1,\tau_2,\dots,\tau_K]$ is the PD operator applied on $x(n)$.

The highest order phase parameter a_P can be estimated from the HAF by performing $(P-1)$ PDs and periodogram maximization:

$$\hat{a}_P=\frac{\arg\max_{\omega}|\mathrm{HAF}(\omega)|}{2^{P-1}P!\Delta^{P-1}\prod_{i=1}^{P-1}\tau_i},$$
$$\mathrm{HAF}(\omega)=\sum_n \mathrm{PD}^{P-1}_{x(n)}[n;\tau_1,\dots,\tau_{P-1}]\exp(-j\omega\Delta n). \tag{3}$$

Once a_P is estimated, lower order parameters and amplitude can be obtained from the dechirped signal $x_d(n)=x(n)\exp(-j\hat{a}_P(n\Delta)^P)$ by repeating the procedure [4].

The PD-based estimation becomes less accurate as the PPS order increases. Each PD generates additional interference terms caused by noise and additional cross-terms in case of several signal components. Moreover, the dechirping procedure causes the error propagation from higher to lower order phase parameters. This effect becomes more emphasized with larger P. The cross-terms could be reduced using the product form of the HAF (PHAF) [5], but the main problems associated with the HAF still remain. Therefore, by reducing the number of PDs, issues associated with the PD implementation are also reduced. However, lower number of PDs increases the computational complexity of estimation due to increased dimensionality of the search space.

The CPF is introduced for the estimation of cubic phase signals ($P=3$) as [1]

$$\mathrm{CPF}(n,\Omega)=\sum_m x(n+m)x(n-m)\exp(-j\Omega(m\Delta)^2). \tag{4}$$

In the absence of noise, the CPF peaks at the second order phase derivative $\Omega(n)=2(3a_3n\Delta+a_2)$ and two highest order phase parameters can be estimated from (4) calculated at $n=0$ and $n=n_1$. The CPF requires lower number of the PDs with respect to the HAF for cubic phase signals and is correspondingly more accurate than the HAF. The CPF is extended for the estimation of higher order PPSs as [3]

$$\mathrm{CPF{-}HAF}(n,\Omega)=\sum_m \mathrm{PD}^{P-3}[n+m;\tau_1,\dots,\tau_{P-3}]\,\mathrm{PD}^{P-3}[n-m;\tau_1,\dots,\tau_{P-3}]\exp(-j\Omega(m\Delta)^2). \tag{5}$$

This approach is referred as the CPF–HAF, and it uses $(P-3)$ PDs to transform a P-th order PPS to a cubic phase signal, whose parameters are in turn estimated by the CPF.

The main problem in the CPF evaluation is transforming the underlying signal by quadratic phase matching sequence $\sum_m[\cdot]\exp(-j\Omega(m\Delta)^2)$ that cannot be evaluated using the FFT. Therefore, the complexity of the CPF-based estimation procedure performed over a search grid of size $O(N)$ is $O(N^2)$, whereas the HAF-based procedure evaluated using the FFT requires $O(N\log N)$ operations. This is the reason for proposing the non-uniform sampled CPF in the next section.

3. Non-uniform sampled CPF

The parameter estimation of higher order PPSs using a non-uniform sampling scheme has been recently proposed in [6]. Here, we apply the same scheme in the CPF definition in order to enable its evaluation by the FFT algorithms.

Maximization of the CPF function is usually performed in two steps. In the first step, a coarse estimate is obtained and that estimate is refined in the second step by searching over the predefined grid around the coarse estimate. The second step can be very computationally demanding. Therefore, several refine search strategies have been proposed for that purpose [10,11]. These strategies are able to obtain very precise estimate by calculating the objective function at several points only. However, these strategies require the evaluation of the objective function by the FFT. Therefore, the CPF evaluation using the FFT would additionally reduce the complexity of the CPF-based estimation procedure.

In this paper, we propose to modify the CPF (4) by substituting m with $m=\sqrt{Ck}$. In that case, the auto-correlation contained in (4) has the following form:

$$x_1(k)=x(n+\sqrt{Ck})x(n-\sqrt{Ck})=A^2\exp\{j(6a_3n\Delta^3Ck+2a_2\Delta^2Ck+2a_3(n\Delta)^3+2a_2(n\Delta)^2+a_1n\Delta+2a_0)\}+\nu_x(n) \tag{6}$$

and the corresponding CPF can be written as

$$\mathrm{NUCPF}_x(n,\Omega)=\sum_k x_1(k)\exp(-j\Omega C\Delta^2k)=\mathrm{FT}\{x_1(k)\}. \tag{7}$$

Relationship (7) gives us the CPF representation by means of the FT. Function (7) will be referred to as the non-uniform sampled CPF (NU-CPF). Note that the NU-CPF peaks at $\Omega(n)=2(3a_3n\Delta+a_2)$, same as the CPF. In order to maintain the same nature of noise after resampling and to achieve high resampling factor, C has to be chosen as $C\approx N/2-|n|$ [6].

The NU-CPF requires the evaluation of the signal $x(n)$ at non-integer time instants, i.e., $n+\sqrt{Ck}$ and $n-\sqrt{Ck}$. Unknown signal values can be obtained using the interpolation procedure which can be described by two steps [6, Section III C.]:

(a) interpolate $x(n)$ by a factor 2 or 4 using some standard interpolation technique to obtain $x_i(n)$;
(b) calculate unknown signal value at arbitrary time