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Brief paper

An effective method to interval observer design for time-varying systems☆

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ABSTRACT

An interval observer for Linear Time-Varying (LTV) systems is proposed in this paper. Usually, the design of such observers is based on monotone systems theory. Monotone properties are hard to satisfy in many situations. To overcome this issue, in a recent work, it has been shown that under some restrictive conditions, the cooperativity of an LTV system can be ensured by a static linear transformation of coordinates. However, a constructive method for the construction of the transformation matrix and the observer gain, making the observation error dynamics positive and stable, is still missing and remains an open problem. In this paper, a constructive approach to obtain a time-varying change of coordinates, ensuring the cooperativity of the observer error in the new coordinates, is provided. The efficiency of the proposed approach is shown through computer simulations.

1. Introduction

The problem of unmeasurable system state estimation is challenging and its solution is required in many engineering applications. The problem of state estimation of systems has many solutions and has been widely investigated in the literature. Popular and well-known observers are mainly based on, for instance, Kalman/H_∞ filtering (Grip, Saberi, & Johansen, 2011; Särkkä, 2007) or Luenberger structure (Barmish & Galimidi, 1986). In situations where external disturbances and noises are assumed bounded without any additional assumption, interval observers can be an appealing alternative approach. Under some assumptions, these observers allow the designer to cope with uncertainties and evaluate the set of admissible values of the state vector, at any time instant.

Several approaches exist for designing interval observers (Bernard & Gouzé, 2004; Jaulin, 2002; Moisan, Bernard, & Gouzé, 2009), for linear systems (Ait Rami, Cheng, & de Prada, 2008; Bernard & Mazenc, 2010; Combastel & Raka, 2011) or when the system exhibits nonlinear behavior (Moisan et al., 2009; Raïssi, Efimov, & Zolghadri, 2012). The design of such observers is based on the monotone systems theory (Ait Rami, Tadeo, & Helmke, 2011; Bernard & Gouzé, 2004; Moisan et al., 2009). This approach has been recently extended to some nonlinear systems using LPV representations with known minorant and majorant matrices (Raïssi, Videau, & Zolghadri, 2010). One of the most restrictive assumptions for the interval observer design is the positivity (Smith, 1995) of the interval estimation error dynamics. It was relaxed for LTI systems in Bernard and Mazenc (2010), Combastel (2013), Combastel and Raka (2011) and Mazenc and Bernard (2011) by using a time-varying change of coordinates. Furthermore, a time-invariant transformation is proposed in Raïssi et al. (2012) to design a closed-loop observer for LTI systems where the transition matrix and the observer gain verify a Sylvester equation. This technique has also been extended to a class of nonlinear systems based on exact

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linearizations. Time-varying systems have been investigated in Efimov, Raïssi, Chebotarev, and Zolghadri (2013) where the case of time-varying transformation for periodic systems is considered and in Efimov, Raïssi, Chebotarev, and Zolghadri (2012) where the observer gain has to ensure stability of the observation error, and a static linear transformation of coordinates is proposed that provides the positivity of the observation error. The main limitation of the technique proposed in Efimov et al. (2013) is that the matrix $D(t) = A(t) - L_{\text{obs}}(t)C(t)$, where $L_{\text{obs}}(t)$ is the observer gain, should belong to a thin domain whose size is proportional to the inverse of the system dimension. Furthermore, no constructive methodology has been provided in Efimov et al. (2013) to prove the existence and to design a similarity transformation making $D(t)$ Metzler in the new coordinates.

The goal of this paper is to design a stable interval observer for LTV systems overcoming the previous limitations. The proposed interval observer is based on a time-varying change of coordinates which has been proposed in earlier works (Zhu & Johnson, 1989a,b, 1991). It should be noted that the proposed methodology does not require any additional assumption with respect to classical observers.

The paper is organized as follows. In Section 2, the problem is formulated and a previous result of an interval observer design for LTV systems is recalled. Section 3 is devoted to a procedure making the transformation of any time-varying matrix into a Metzler matrix. The result given in Section 3 is then used to design an interval observer for LTV systems in Section 4. Section 5 shows the efficiency of the interval observer through numerical simulations. To emphasize the improvement, a comparison with previous results reported in Efimov et al. (2013) and Thabet, Raïssi, Combastel, and Zolghadri (2013) is given.

2. Notations and problem statement

A square matrix $A = (A_{ij}) \in \mathbb{R}^{n\times m}$ is said to be Metzler if $A_{ij} \geq 0,\ \forall i \neq j$. For two vectors $x_1, x_2 \in \mathbb{R}^n$ or matrices $A_1, A_2 \in \mathbb{R}^{n\times n}$, the relations $x_1 \leq x_2$ and $A_1 \leq A_2$ are understood elementwise. The relation $P \prec 0$ ($P \succ 0$) means that the matrix $P \in \mathbb{R}^{n\times n}$ is negative (positive) definite.

Lemma 1 (*Smith, 1995*). *Given a non-autonomous system described by $\dot{x}(t) = Ax(t) + B(t)$ where A is a Metzler matrix and $B(t) \geq 0$. Then, $x(t) \geq 0,\ \forall t > 0$ provided that $x(0) \geq 0$.*

Note that the result of Lemma 1 is also valid for time-varying systems (i.e. $A(t)$ is time-varying). Now, consider an LTV system described by:

$$\begin{cases} \dot{x}(t) = A(t)x(t) + f(t) \\ y(t) = C(t)x(t) + \varphi(t) \\ x(0) \in [\underline{x}(0), \overline{x}(0)] \\ \quad \forall t,\ f(t) \in [\underline{f}(t), \overline{f}(t)] \subset \mathbb{R}^n,\ \varphi(t) \in [\underline{\varphi}(t), \overline{\varphi}(t)] \subset \mathbb{R}^p \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n, f(t) \in \mathbb{R}^n, y(t) \in \mathbb{R}^p$ and $\varphi(t) \in \mathbb{R}^p$ are respectively the state vector, an unknown but bounded input, the output vector and a bounded noise. The goal is to design an interval observer for systems described by (1).

Assumption 1. There exist bounded matrix functions $L_{\text{obs}} : \mathbb{R} \to \mathbb{R}^{n\times p}$, $M : \mathbb{R}_+ \to \mathbb{R}^{n\times n}$, $M(\cdot) = M(\cdot)^T \succ 0$ such that for all $t \geq 0$,

$$\begin{cases} \dot{M}(t) + D(t)^T M(t) + M(t)D(t) \prec 0, \\ D(t) = A(t) - L_{\text{obs}}(t)C(t). \end{cases}$$

Assumption 1 is a conventional requirement for LTV systems (Amato, Pironti, & Scala, 1996). Under this assumption, the observer gain $L_{\text{obs}}(t)$ and the matrix function $M(t)$ are such that the stability of the LTV system $\dot{x}(t) = D(t)x(t)$ can be proven by taking $V(t) = x(t)^T M(t)x(t)$ as Lyapunov function. It determines the output stabilization conditions of the system dynamics (1) which can be rewritten as:

$$\begin{cases} \dot{x}(t) = D(t)x(t) + \tilde{\phi}(t) \\ y(t) = C(t)x(t) + \varphi(t) \end{cases} \tag{2}$$

where $\tilde{\phi}(t) = f(t) - L_{\text{obs}}(t)\varphi(t) + L_{\text{obs}}(t)y(t)$. Linear Parameter-Varying or polytopic system results (Anstett, Millrioux, & Bloch, 2009; Bara, Daafouz, Ragot, & Kartz, 2000) can be used to compute an observer gain $L_{\text{obs}}(t)$ satisfying Assumption 1. In addition, if the matrix $D(t) = A(t) - L_{\text{obs}}(t)C(t)$ is Metzler, an interval observer for the LTV system (2) can be easily designed. Nevertheless, the Metzler condition is not usually satisfied without applying some model transformations. This problem has been investigated in recent work (Efimov et al., 2013) where the goal was to find a time-invariant change of coordinates and an observer gain in order to obtain a positive observation error at each time. The design of the gain and the existence of the static transition matrix ensuring the Metzler property remains a difficult task. Assumption 2 (Assumption 4 in Efimov et al., 2013) was used to design interval observers.

Assumption 2. Let $D(t) \in \Xi$ for all $t \geq 0$, $\Xi = \{D \in \mathbb{R}^{n\times n} : D_a - \Delta \leq D \leq D_a + \Delta\}$ for some $D_a^T = D_a \in \mathbb{R}^{n\times n}$ and $\Delta \in \mathbb{R}_+^{n\times n}$. Let for some constant $\mu > n\|\Delta\|_{\max}$ (where $\|\Delta\|_{\max} = \max_{i=\overline{1,n}, j=\overline{1,n}} |\Delta_{i,j}|$ the elementwise maximum norm) and a diagonal matrix $\Upsilon \in \mathbb{R}^{n\times n}$ the Metzler matrix $R = \mu E_n - \Upsilon$, where $E_n \in \mathbb{R}^{n\times n}$ denotes the matrix with all elements equal to 1, have the same eigenvalues as the matrix D_a.

Note that the case of $\Delta = 0$ corresponds to LTI systems for which several solutions exist (Combastel, 2013; Mazenc & Bernard, 2011; Raïssi et al., 2012). Under Assumption 2, (Efimov et al., 2013) shows that there is an orthogonal matrix $S \in \mathbb{R}^{n\times n}$ such that the matrices $S^T D(t) S$ are Metzler for all $D(t) \in \Xi$. By introducing the new state variable $z = S^T x$, (1) can be rewritten in the new coordinates:

$$\dot{z} = S^T A(t) S z + \phi(t),$$

where $\phi(t) = S^T f(t)$. The proposed interval observer for the system (1) in the new coordinates is:

$$\begin{cases} \dot{\underline{z}} = S^T D(t) S \underline{z} + \underline{\phi}(t) + \underline{\Psi}(t) + K_{\text{obs}}(t) y \\ \dot{\overline{z}} = S^T D(t) S \overline{z} + \overline{\phi}(t) + \overline{\Psi}(t) + K_{\text{obs}}(t) y \end{cases} \tag{3}$$

where $\underline{\phi}(t) = (S^+)^T \underline{f}(t) - (S^-)^T \overline{f}(t)$, $\overline{\phi}(t) = (S^+)^T \overline{f}(t) - (S^-)^T \underline{f}(t)$, $K_{\text{obs}} = S^T L_{\text{obs}}(t)$, $\underline{\Psi}(t) = K_{\text{obs}}^-(t)\underline{\varphi} - K_{\text{obs}}^+(t)\overline{\varphi}$, $\overline{\Psi}(t) = K_{\text{obs}}^-(t)\overline{\varphi} - K_{\text{obs}}^+(t)\underline{\varphi}$. Given a matrix $N \in \mathbb{R}^{m\times n}$, N^+ and N^- are defined as: $N^+ = \max\{0, N\}$, $N^- = \max\{0, -N\}$. Then, S^+, S^-, $K_{\text{obs}}^+(t)$ and $K_{\text{obs}}^-(t)$ can be deduced. In the original coordinates, applying Lemma 2 given below to the relation $x = Sz$, the bounds of the state vector x are given by:

$$\underline{x} = S^+ \underline{z} - S^- \overline{z}, \qquad \overline{x} = S^+ \overline{z} - S^- \underline{z}. \tag{4}$$

Lemma 2 (*Efimov et al., 2013*). *Let $x \in \mathbb{R}^n$ be a vector variable, $\underline{x} \leq x \leq \overline{x}$ for some $\underline{x},\ \overline{x} \in \mathbb{R}^n$, and $S \in \mathbb{R}^{m\times n}$ be a matrix, then*

$$S^+ \underline{x} - S^- \overline{x} \leq Sx \leq S^+ \overline{x} - S^- \underline{x}. \tag{5}$$

According to Assumption 2, the main limitation of the technique proposed in Efimov et al. (2013) is that the matrix $D(t)$ should belong to a thin domain whose size is proportional to the inverse of the system dimension ($\|\Delta\|_{\max} < \frac{\mu}{n}$). Then, the bigger the system dimension is, the thinner the domain enclosing $D(t)$ must