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Brief paper

Design of delta–sigma modulators via generalized Kalman–Yakubovich–Popov lemma[☆]

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ABSTRACT

This paper is concerned with the design of delta–sigma modulators via the generalized Kalman–Yakubovich–Popov lemma. The shaped noise transfer function (NTF) is assumed to have infinite impulse response, and the optimization objective is minimizing the maximum magnitude of the NTF over the signal frequency band. By virtue of the GKYP lemma, the optimization of an NTF is converted into a minimization problem subject to quadratic matrix inequalities, and then an iterative algorithm is proposed to solve this alternative minimization problem. Each iteration of the algorithm contains linear matrix inequality constraints only and can be effectively solved by the existing numerical software packages. Moreover, specifications on the NTF zeros are also integrated in the design method. A design example demonstrates that the proposed design method has an advantage over the benchmark one in improving the signal-to-noise ratio.

1. Introduction

Analog-to-digital (A/D) and digital-to-analog (D/A) data converters are the indispensable part of most electronic systems. As the interface between the digital signal world and the real analog world, they determine whether and how much the conversion can correctly keep the important information of signals, meanwhile suppressing undesirable noises. To improve the resolution of A/D converters, it has been long recognized that the framework of delta–sigma ($\Delta\Sigma$) modulators is an effective scheme (Oppenheim, Schafer, & Buck, 1998). Even under a coarse quantizer, the $\Delta\Sigma$ modulator scheme combined with the oversampling technique can provide a very high resolution (Aziz, Sorensen, & der Spiegel, 1996). Especially, with the increasing requirement on converters of high quality, research on $\Delta\Sigma$ modulators has attracted considerable attention during the past decades, and applications can also be found in many areas related to digital signal processing (see the related literature in Part I of Table 1).

The central task of designing $\Delta\Sigma$ modulators is *noise shaping* (Schreier & Temes, 2005). By virtue of optimization theory and the computer-aided design technique, some design methods have been developed for noise transfer function (NTF) shaping (see Part II of Table 1). A typical off-the-shelf tool is The Delta–Sigma Toolbox (Schreier, 2009), for which, the objective of NTF shaping is to minimize the integral of the squared magnitude of the NTF over the signal frequency band. Signal processing and systems theory are two areas tightly related to each other (Li, Jing, & Karimi, 2014; Yang, Liu, Shi, Thomas, & Basin, 2014). From the perspective of systems theory, $\Delta\Sigma$ modulators can be regarded as a special class of systems, and thus many results in control and systems theory are potentially beneficial for $\Delta\Sigma$ modulator design. For instance, the Bounded Real lemma exposes that the H_∞ norm of a transfer function can be characterized by a linear matrix inequality (LMI) (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994), which has been employed for describing the magnitude specification of the NTF in terms of a convex constraint (Nagahara, Wada, &

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Table 1
A brief summary of the related literature.

Part I: applications	
Wireless communication	Schoofs et al. (2007)
Control systems	Azuma and Sugie (2008a,b)
Audio amplifiers	Dooper and Berkhout (2012)
Sensing and measuring	Acero et al. (2007)
Part II: design methods	
Average gain optimization	Callegari and Bizzarri (2013) and Schreier (2009)
Peak gain optimization	Nagahara et al. (2006), Osqui et al. (2007) and Nagahara and Yamamoto (2012)
Advanced control theory	Quevedo and Goodwin (2005) and Yu (2006)
Others	Azuma and Sugie (2008a,b)

Yamamoto, 2006). Another promising result recently developed in control theory is the generalized Kalman–Yakubovich–Popov (GKYP) lemma (Iwasaki & Hara, 2005), based on which, some new developments on $\Delta\Sigma$ modulator design also have been reported in the literature (Nagahara et al., 2006; Osqui, Roozbehani, & Megretski, 2007). In Osqui et al. (2007), an upper bound of the low frequency average power of the reconstruction error was found, and a numerical method in light of the GKYP lemma was then proposed for approximately minimizing the upper bound. However, the derived upper bound and the design algorithm in Osqui et al. (2007) were *only* applicable for binary, low-pass $\Delta\Sigma$ modulators; moreover, some optional parameters were chosen empirically in the design algorithm there. Via the GKYP lemma, a recent paper (Nagahara & Yamamoto, 2012) proposed a min–max approach to NTF optimization. This approach minimizes the *worst-case* gain of the NTF over the signal frequency band and is shown to be able to improve the overall SNR of $\Delta\Sigma$ modulators. Due to the advantage of the GKYP lemma, the approach in Nagahara and Yamamoto (2012) avoids the selection of a weighting function that is used in Nagahara et al. (2006).

The method proposed in Nagahara and Yamamoto (2012) can design finite impulse response (FIR) NTF *only*, while for NTFs with infinite impulse response (IIR), it is *inapplicable*. This limitation of the method in Nagahara and Yamamoto (2012) motivates the research in this paper, that is, designing IIR NTFs by making use of the GKYP lemma. Compared with the results in Nagahara and Yamamoto (2012), the current investigation is not trivial. On one hand, an IIR NTF with a much lower order can achieve a comparable performance as an FIR one. In fact, FIR NTFs are only a special case of IIR NTFs. On the other hand, the approach to shaping FIR NTFs in Nagahara and Yamamoto (2012) *cannot* be extended to the IIR case, and solving or testing the conditions derived for the IIR case is much more involved than that for the FIR case.

In this paper, we focus on IIR NTF shaping for $\Delta\Sigma$ modulators. To this end, the min–max strategy in Nagahara and Yamamoto (2012) is employed as the optimizing objective. A new condition is first derived for characterizing the desired frequency-domain shape of an NTF. Since this condition, in the quadratic matrix inequality (QMI) form, cannot be directly solved, we further handle it by an iterative algorithm. In each step, it only needs to test several LMI constraints, which can be completed by the existing numerical software packages (Gahinet, Nemirovskii, Laub, & Chilali, 1995). We also discuss how to choose the empirical feasible initialization condition and incorporate the specifications on the NTF zeros in the same framework. In the presented illustrative design example, we compare the proposed $\Delta\Sigma$ modulator with the benchmark one designed by The Delta–Sigma Toolbox (Schreier, 2009), showing that the proposed design method can improve both of the worst-case SNR and the average SNR. Some preliminary results in this paper can be found in Li, Gao, and Yu (2013).

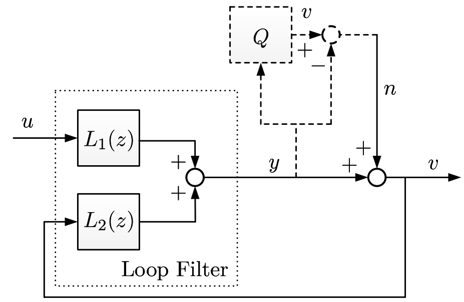

Fig. 1. General structure of a $\Delta\Sigma$ modulator with loop filter $[L_1(s), L_2(s)]$, and quantizer Q.

Notation: The superscripts "-1", "T" and "$*$" stand for inverse, transpose, and conjugate transpose of a matrix, respectively. The notation $P > 0$ means that matrix P is positive definite. $\mathbf{I}$ denotes an identity matrix with appropriate dimension.

2. Main results

2.1. $\Delta\Sigma$ modulator and NTF shaping

Detailed description on $\Delta\Sigma$ modulators can be found in Schreier and Temes (2005), and some related basic concepts are briefly introduced here so as to bring out the problem. Consider a $\Delta\Sigma$ modulator shown in Fig. 1, where $u(k)$ is a discrete-time scalar-valued input signal, $[L_1(z), L_2(z)]$ is the linear loop filter, and Q denotes a general quantizer. Note that $n(k) = Q[y(k)] - y(k)$ and $v(k) = n(k) + y(k) = Q[y(k)]$. Hence, $y(k)$ and $v(k)$ are the discrete-time output signals before and after quantization, respectively, while $n(k)$ is the quantization error. The basic function of a $\Delta\Sigma$ modulator is to convert $u(k)$ (e.g., analog signal) into $v(k)$ (e.g., digital signal) for other uses.

In the linear part, quantized output $v(k)$ can be denoted in terms of inputs $u(k)$ and $n(k)$, that is,

$$V(z) = T_{\mathrm{STF}}(z)U(z) + T_{\mathrm{NTF}}(z)N(z)$$

where $T_{\mathrm{STF}}(z)$ and $T_{\mathrm{NTF}}(z)$ are referred to as the signal transfer function (STF) and the NTF, and $V(z)$, $U(z)$ and $N(z)$ are the z-domain representation of signals $v(k)$, $u(k)$ and $n(k)$, respectively. Corresponding to Fig. 1, it is easy to obtain that

$$T_{\mathrm{STF}}(z) = \frac{L_1(z)}{1 - L_2(z)}, \qquad T_{\mathrm{NTF}}(z) = \frac{1}{1 - L_2(z)}. \tag{1}$$

Hence, loop filters $L_1(z)$ and $L_2(z)$ can be parameterized by STF $T_{\mathrm{STF}}(z)$ and NTF $T_{\mathrm{NTF}}(z)$. As is known, noise shaping, or more exactly, NTF shaping, is the central task of designing $\Delta\Sigma$ modulators (Schreier & Temes, 2005). An appropriate NTF should satisfy the following three basic requirements.

Realizability: To guarantee physical realizability of the modulator, at least one clock-period delay must be contained in the loop formed by filter $L_2(z)$ and quantizer Q; otherwise, $y(k)$ in one sample would pass through Q and $L_2(z)$ instantly, making $y(k)$ continuously vary during the same sampling period. In view of Fig. 1, this delay should be in $L_2(z)$, implying that $L_2(z)$ must be *strictly proper*. Mathematically, this means $L_2(\infty) = 0$. Recasting this requirement to $T_{\mathrm{NTF}}(z)$, one sees that $T_{\mathrm{NTF}}(z)$ must satisfy

$$T_{\mathrm{NTF}}(\infty) = \frac{1}{1 - L_2(\infty)} = 1 \tag{2}$$

which is an elementary restriction when designing $T_{\mathrm{NTF}}(z)$. For more detailed interpretation on this restriction, please refer to Schreier and Temes (2005, pp. 95–97), and Nagahara and Yamamoto (2012) also provides an interpretation from a well-posedness perspective.