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Brief paper

Design of Marx generators as a structured eigenvalue assignment☆

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ABSTRACT

We consider the design problem for a Marx generator electrical network, a pulsed power generator. We show that the components design can be conveniently cast as a structured real eigenvalue assignment with significantly lower dimension than the state size of the Marx circuit. Then we present two possible approaches to determine its solutions. A first symbolic approach consists in the use of Gröbner basis representations, which allows us to compute all the (finitely many) solutions. A second approach is based on convexification of a nonconvex optimization problem with polynomial constraints. We also comment on the conjecture that for any number of stages the problem has finitely many solutions, which is a necessary assumption for the proposed methods to converge. We regard the proof of this conjecture as an interesting challenge of general interest in the real algebraic geometry field.

1. Introduction

Electrical pulsed power generators have been studied from the 1920s with the goal to provide high power electrical pulses by way of suitable electrical schemes that are slowly charged and then, typically by the action of switches, are rapidly discharged to provide a high voltage impulse or a spark (see, e.g., Bluhm & Rusch, 2006). Marx generators (see Bluhm & Rusch, 2006, Section 3.2 or Carey & Mayes, 2002 for an overview) were originally described by E. Marx in 1924 and correspond to circuits enabling generation of high voltage from lower voltage sources. While many schemes have been proposed over the years for Marx generators, a recent understanding of certain compact Marx generators structures (Buchenauer, 2010) reveals that their essential behavior can be well described by a suitable *LC* ladder network where certain components should be designed in order to guarantee a suitable resonance condition. In turn, such a resonance condition is known to lead to a desirable energy transfer throughout the circuit and effective voltage multiplication which can then be used for pulsed power generation.

This paper addresses the mathematical problem of designing the lumped components of the compact Marx generator circuit well described in Buchenauer (2010) and represented in Fig. 1. We show here that this design can be cast as a structured real eigenvalue assignment problem for a linear system only depending on the number of its stages.

The problem of static output feedback pole (or eigenvalue) assignment for linear systems has been largely studied in the 1990s, see Rosenthal and Willems (1999) for a survey. In its simplest form, it can be stated as follows: given matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and a monic polynomial $q(s) \in \mathbb{R}[s]$ of degree n, find an $m \times p$ real matrix F such that $\det(sI_n - A - BFC) = q(s)$ where I_n denotes the identity matrix of size n. This

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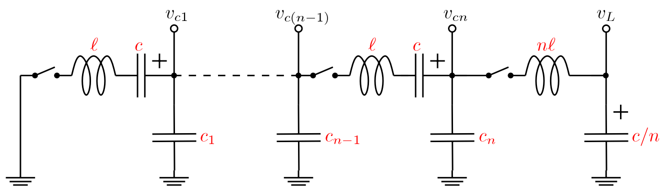

Fig. 1. The passive circuit used as a Marx generator.

problem has generically a solution if $mp > n$, and it has generically no solution if $mp < n$. The situation $mp = n$ is much more subtle. For this situation, it was proved in Eremenko and Gabrielov (2002a,b) that the pole placement map from the feedback matrix F to the characteristic polynomial $q(s)$ is generically not surjective. It means that there is a non-empty open subset of real matrices A, B, C for which there exist open sets of pole configurations symmetric w.r.t. the real axis which cannot be assigned by any real feedback. In this paper, we do not consider static output feedback pole assignment in the form just described, but in a structured form. The number of degrees of freedom (number of entries in the feedback matrix) is equal to n, the number of poles to be assigned, so it bears similarity with the difficult case $mp = n$ of static output feedback pole assignment described above.

Preliminary results along the direction of this paper were presented in Zaccarian et al. (2009), where Laplace domain descriptions were used and where a result roughly corresponding to one of the key components of our proof was stated without any proof. Here, in addition to proving that result, we provide a more complete statement establishing sufficient conditions for the desired pulsed power generation. Moreover, we adopt a state-space representation that allows to provide an elegant and compact proof of our main result. Finally, an important contribution of this paper consists in the two solution methods outlined above. The symbolic one allows us to enumerate all the possible component selections ensuring the resonant conditions for circuits with up to $n = 6$ stages. The numerical one allows us to compute the so-called "regular solution" (characterized in Section 3.2) for the more convoluted cases $n = 7$ and $n = 8$. Some technical proofs are omitted due to space constraints but can be found in Galeani, Henrion, Jacquemard, and Zaccarian (2013).

Notation. Given a square matrix A, $\sigma(A)$ denotes its spectrum, i.e., the set of its complex eigenvalues. Given a vector $f \in \mathbb{R}^n$, $\mathbb{Q}[f]$ denotes the set of all polynomials with rational coefficients in the indeterminates f.

2. Marx generator design

2.1. Circuit description and problem statement

We consider the Marx generator network shown in Fig. 1 consisting in n stages (and $n + 1$ loops) where, disregarding the two rightmost components of the figure, each one of the n stages consists of (1) an upper branch with a capacitor and an inductor and (2) a vertical branch with a capacitor only. Following (Antoun, 2006; Buchenauer, 2010; Zaccarian et al., 2009), we assume that all the capacitors and inductors appearing in the upper branches are the same (corresponding to some fixed positive reals c and ℓ). We will call these capacitors "storage capacitors" in the sequel, for reasons that will become clear next. The design problem addressed here is the selection of the vertical capacitors, which are exactly n, where n is the number of stages of the Marx circuit. We will call these capacitors "parasitic capacitors" due to their position resembling that of parasitic capacitors in transmission lines. Despite their name, the parasitic capacitors c_i, $i = 1, \ldots, n$ are not necessarily arising from any parasitic effects and their values will be selected in such a way to ensure a suitable resonance condition as clarified next.

Following (Antoun, 2006; Buchenauer, 2010; Zaccarian et al., 2009), the inductance and capacitor appearing in the rightmost loop take the values $n\ell$ and c/n, respectively. We call this capacitor the "load capacitor". This selection preserves the resonance property (so that the product of any adjacent capacitor/inductor pairs is always ℓc) in addition to ensuring that the load capacitor is n times larger than each one of the storage capacitors. The problem addressed in this paper (resembling that one tackled in Antoun, 2006, Buchenauer, 2010 and Zaccarian et al., 2009) is the following.

Problem 1. Consider the circuit in Fig. 1 for given n and certain values of c and ℓ. Select positive values $c_i > 0$, $i = 1, \ldots, n$ of the parasitic capacitors and a time $T > 0$ such that, initializing at $t = 0$ all the storage capacitors with the same voltage $v(0) = v_\circ$ and starting from zero current in all the inductors and zero voltage across the parasitic capacitors and the load capacitor, the circuit response is such that at $t = T$ all voltages and currents are zero except for the voltage across the load capacitor.

2.2. Solution via structured eigenvalue assignment

In this section we show that a solution to Problem 1 can be determined from the solution of a suitable structured eigenvalue assignment problem involving a tridiagonal matrix $B \in \mathbb{R}^{n\times n}$ with elements $B_{i,i} = 2$, $B_{i,i+1} = B_{i+1,i} = -1$ for $i = 1, \ldots, n-1$, $B_{n,n} = 1 + n^{-1}$, and an arbitrary set of even harmonics of the fundamental frequency $\omega_0 = \sqrt{(\ell c)^{-1}}$ to be assigned to the circuit. The following is the main result of this paper, and its proof is given in Section 5.3.

Theorem 1. *Consider any set of n distinct positive even integers $\alpha = (\alpha_1, \ldots, \alpha_n)$, the matrix B as above and any positive definite real diagonal solution $F = \operatorname{diag}(f_1, \ldots, f_n)$ to the structured eigenvalue assignment problem*

$$\sigma(BF) = \{\alpha_1^2 - 1, \ldots, \alpha_n^2 - 1\}. \qquad (1)$$

Then for any value of c, the selection $c_i = c/f_i$, $i = 1, \ldots, n$, solves Problem 1 for all values of ℓ with $T = \frac{\pi}{\sqrt{\ell c}}$.

Theorem 1 shows that a solution to Problem 1 can be determined by solving an eigenvalue assignment problem with decentralized feedback (because matrix F is diagonal). Note that the structure in this problem arises naturally from the physical nature of the circuit under consideration and does not arise from some simplifying assumptions on the circuit behavior. A generalized version of this problem was studied in Wang (1994) (indeed, using the notations there, problem (1) is obtained by setting $r = n$, $C = I_n$, $m_i = p_i = 1$, $i = 1, \ldots, n$). It is shown in Wang (1994) that generic pole assignment depends on the dimension of a product Grassmannian, see Wang (1994, Eq. (17)). In our case it is equal to $n!$ which is always even, and from Wang (1994, Theorem 4.2) it follows that generic pole assignment cannot be achieved. A geometric condition that ensures generic pole assignment is given in Wang (1994, Prop. 4.2) but we do not know whether this condition can be checked computationally. In any case it is an evidence that the question of existence of a real solution to our problem does not appear to be trivial.

The following result, proved in Galeani et al. (2013), shows that any solution to (1) is physically implementable as it corresponds to positive values of the parasitic capacitors.

Lemma 1. *Any solution F to the structured eigenvalue assignment (1) (in Theorem 1) satisfies $F > 0$.*

2.3. Two equivalent formulations

Selecting the diagonal entries $f = [f_1 \cdots f_n]^T$ to solve (1) amounts to solving a finite set of n equations (polynomial in the