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Stabilization and robust H_∞ control for sector-bounded switched nonlinear systems[☆]

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ABSTRACT

This paper presents stability analysis and robust H_∞ control for a particular class of switched systems characterized by nonlinear functions that belong to sector sets with arbitrary boundaries. The sector boundaries can have positive and/or negative slopes, and therefore, we cover the most general case in our approach. Using the special structure of the system but without making additional assumptions (e.g. on the derivative of the nonlinear functions), and by proposing new multiple Lyapunov function candidates, we formulate stability conditions and a control design procedure in the form of matrix inequalities. The proposed Lyapunov functions are more general than the quadratic functions previously proposed in the literature, as they incorporate the nonlinearities of the system and hence, lead to less conservative stability conditions. The stabilizing switching controllers are designed through a bi-level optimization problem that can be efficiently solved using a combination of a convex optimization algorithm and a line search method. The proposed optimization problem is achieved using a special loop transformation to normalize the arbitrary sector bounds and by other linear matrix inequalities (LMI) techniques.

1. Introduction

Switched systems arise in cases where several dynamical models are required to represent a system due to e.g. uncertainty in parameters, or specific applications that utilize switching between controllers in order to achieve a higher performance (Aleksandrov, Chen, Platonov, & Zhang, 2011; Geromel, Deaecto, & Daafouz, 2013; Hu, Ma, & Lin, 2008; Liberzon, 2003).

In this work, we study a special case of switched systems comprising a set of nonlinear subsystems. In each subsystem, the evolution of states is governed by linear combinations of nonlinear state-dependent functions. Furthermore, a switching signal determines the active subsystem at each time instant. The nonlinear functions are assumed to belong to sector sets with arbitrary (positive or negative, and possibly asymmetric) slopes for the sector boundaries. Thus, in the non-switched case, we cover more general cases of nonlinear functions compared e.g. to the Lure' type systems studied by Castelan, Tarbouriech, and Queinnec (2008), Gonzaga, Jungers, and Daafouz (2012) and to the nonlinear systems that admit diagonal-type Lyapunov functions investigated by Aleksandrov et al. (2011) and Kazkurewicz and Bhaya (1999).

Our paper contains 3 main contributions with respect to the state-of-the-art: (1) inclusion of sector bounds with arbitrary slopes for nonlinear functions (moreover, the nonlinear functions are no longer required to have an unbounded integral), (2) stability analysis for this class of switched systems under arbitrary switching using a less conservative approach based on multiple Lyapunov functions and the concept of average dwell time, (3) stabilization and robust disturbance attenuation of these systems using a bi-level convex optimization problem.

For stability analysis under arbitrary switching, we propose a family of Lyapunov functions that incorporate both quadratic functions of states and also the integrals of nonlinearities in the subsystems. Since the proposed Lyapunov candidate functions are general and include the nonlinear dynamics, this choice in general will lead to less conservative stability conditions compared to e.g. the choice of quadratic functions (see Castelan et al., 2008, Gomes da Silva, Castelan, & Eckhard, 2013 for a specific non-switched case). Based on the concept of average dwell-time (Hespanha, 2004), which allows fast switching occasionally, we formulate the feasibility problem based on matrix inequalities that are nonlinear in a single

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scalar variable, in order to find a lower bound for the average dwell time to ensure asymptotic stability.

Next, we investigate the stabilization problem for the given class of switched systems in case of unstable modes and disturbances. Combining the proposed Lyapunov functions and their derivatives in order to obtain a single expression that can be used to design controllers is challenging. This is because the Lyapunov functions include the integrals of nonlinear functions while in the time derivative of the Lyapunov functions, the nonlinear functions appear explicitly. Using a transformation to normalize the sector bounds and congruence transformations in order to re-arrange the matrix inequalities into linear ones, design conditions are formulated in the form of a bi-level optimization problem with high-level problem non-convex only in a single scalar variable, and a convex low-level problem. Hence, the overall problem can be efficiently solved using a line search method along with LMIs feasibility checking.

In the following, we first present the problem statement and next, the stability conditions for the system under arbitrary switching. The design of robust stabilizing controllers is presented next and finally, the performance of the proposed scheme is shown using an example.

2. Problem statement and background

Consider the following switched nonlinear system:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + E_{\sigma(t)}f\big(x(t)\big) + H_{\sigma(t)}\omega(t), \tag{1}$$

$$u(t) = K_{\sigma(t)}x(t) + F_{\sigma(t)}f(x), \tag{2}$$

$$y(t) = C_{\sigma(t)}g\big(x(t)\big), \tag{3}$$

with $x = (x_1, \dots, x_n)^{\mathrm{T}}$ the state vector, $u \in \mathbb{R}^{n_u}$ the control input, $\omega \in \mathbb{R}^{n_\omega}$ the exogenous input, $y \in \mathbb{R}^{n_y}$ the output, and $f : \mathbb{R}^n \to \mathbb{R}^n : x_i \to f_i(x_i)$, $g : \mathbb{R}^n \to \mathbb{R}^n : x_i \to g_i(x_i)$ nonlinear vector functions. Moreover, the switching signal σ is defined as a piecewise constant function, $\sigma(\cdot) : [0, +\infty) \to \{1, \dots, N\}$.

Assumption 1. The scalar functions f_i are continuous and belong to the class $\mathcal{S}_{c1}$ defined as follows:

$$\mathcal{S}_{c1} = \{\phi : \mathbb{R} \to \mathbb{R} |\, \big(\phi(\zeta) - \alpha\zeta\big)\big(\phi(\zeta) - \beta\zeta\big) \le 0,\ \phi(0) = 0, \\ \forall\zeta \in \mathbb{R},\ \alpha, \beta \in \mathbb{R}, \alpha < \beta\}. \tag{4}$$

Note that the functions f_i are neither required to lie only in the 1st and the 3rd quadrant as in Gonzaga et al. (2012), nor to have unbounded integrals as in Aleksandrov et al. (2011) and Kazkurewicz and Bhaya (1999).

Assumption 2. The scalar functions g_i are continuous and belong to the class $\mathcal{S}_{c2}$ defined as follows:

$$\mathcal{S}_{c2} = \{\psi : \mathbb{R} \to \mathbb{R} |\, \exists\delta \text{ such that } |\psi(\zeta)| \le \delta|\zeta|,\ \forall\zeta \in \mathbb{R}\}. \tag{5}$$

In fact, $\mathcal{S}_{c2}$ is a special case of the class $\mathcal{S}_{c1}$ and functions that belong to the class $\mathcal{S}_{c2}$ are bounded within a symmetric convex double cone with the origin as apex. Moreover, the nonlinear functions in system (1) can also be considered as state-dependent disturbances. Therefore, specific applications in which these type of disturbances affect the system, can be treated with our proposed analysis and control tools.

For the non-switched case of system (1), Ionsian and Susya (1991) proved that a Lyapunov function formulated as:

$$V(x) = x^{\mathrm{T}}Px + \sum_{i=1}^{n} \lambda_i \int_0^{x_i} f_i(\xi)\mathrm{d}\xi, \tag{6}$$

always exists provided that A is Hurwitz, $x_i f_i(x_i) \ge 0$, $\forall i$ and E has nonnegative off-diagonal elements. However, stability of a composed switched system cannot be concluded from the stability of subsystems (Liberzon, 2003). Stability under arbitrary switching for system (1) with $A_i = 0$ using a common Lyapunov function of the form (6) but without the quadratic term is proposed by Aleksandrov et al. (2011). However, extension of the results obtained by Aleksandrov et al. (2011) to our more general model (1)–(3) and more important, to the robust stabilization problem is not possible. This is mainly because we need to combine and compare the Lyapunov functions and their derivatives in order to compose a stabilizing control law and this is not feasible with the current formulation of the Lyapunov function (6) (due to the integral of the nonlinearities). One solution is to use only quadratic functions of states. However, this choice would increase the conservatism in the stability analysis. Therefore, in the following, we use a different Lyapunov function that still contains the nonlinearities in the model and meanwhile, is extendable for the design of stabilizing switching laws. In the first stage, we propose a less conservative approach (compared to the common Lyapunov function method) for stability under arbitrary switching, using the concept of dwell time.

3. Stability analysis under arbitrary switching with dwell time constraint

For the switched system (1) with $u(t), \omega(t) = 0\ \forall t$, the following set of Lyapunov functions is proposed:

$$V_\ell(x) = x^{\mathrm{T}}P_\ell x + 2\sum_{i=1}^{n} \lambda_i^{(\ell)} \int_0^{x_i} f_i(\xi)\mathrm{d}\xi, \quad \ell = 1, \dots, N. \tag{7}$$

With this choice, the functions f_i are not required to have an unbounded integral (in contrast to the Lyapunov function in Aleksandrov et al., 2011). The following theorem provides sufficient conditions for exponential stability of (1) using the concepts of multiple Lyapunov functions (Colaneri, Geromel, & Astolfi, 2008) and the average dwell time (Hespanha, 2004). Note that asymptotic stability of all subsystems is a necessary condition here.

Theorem 3. *Consider the system (1) with Assumption 1. Suppose there exist positive matrices $\Lambda_\ell = \mathrm{diag}\{\lambda_i^{(\ell)}\}$, symmetric matrices P_ℓ, positive diagonal matrices $\mathcal{T}_\ell$, for $\ell = 1, \dots, N$, and a positive scalar ε, such that:*

$$\begin{bmatrix} P_\ell A_\ell + A_\ell^{\mathrm{T}}P_\ell + \varepsilon(P_\ell + \Lambda_\ell\mathcal{D}_\beta) - \mathcal{T}_\ell\mathcal{D}_\alpha\mathcal{D}_\beta & \star \\ E_\ell^{\mathrm{T}}P_\ell + \Lambda_\ell A_\ell + \frac{1}{2}\mathcal{T}_\ell(\mathcal{D}_\alpha + \mathcal{D}_\beta) & \Lambda_\ell E_\ell + E_\ell^{\mathrm{T}}\Lambda_\ell - \mathcal{T}_\ell \end{bmatrix} < 0, \tag{8}$$

$$P_\ell + \Lambda_\ell\mathcal{D}_\alpha > 0, \quad \forall\ell \in \{1, \dots, N\}, \tag{9}$$

with $\mathcal{D}_\alpha = \mathrm{diag}\{\alpha_i\}$, $\mathcal{D}_\beta = \mathrm{diag}\{\beta_i\}$. System (1) with $u, \omega \equiv 0$ is globally exponentially stable under arbitrary switching, if the average dwell time of consecutive switching instants for any arbitrary interval (t_0, t) is bounded by:

$$T_{\mathrm{D}}(t_0, t) \ge \frac{1}{\varepsilon}\log\left(\max_{j,\ell\in\{1,\dots,N\}} \frac{b_{\max,j}}{a_{\min,\ell}}\right), \quad \forall t > t_0, \tag{10}$$

where $a_{\min,\ell}$ denotes the smallest singular value of $(P_\ell + \Lambda_\ell\mathcal{D}_\alpha)$ and $b_{\max,j}$ is the largest singular value of $(P_j + \Lambda_j\mathcal{D}_\beta)$.

Proof. Since $\lambda_i^{(\ell)} > 0$, we have:

$$\sum_{i=1}^{n} \lambda_i^{(\ell)} \int_0^{x_i} \alpha_i\xi\mathrm{d}\xi \le \sum_{i=1}^{n} \lambda_i^{(\ell)} \int_0^{x_i} f_i(\xi)\mathrm{d}\xi \\ \le \sum_{i=1}^{n} \lambda_i^{(\ell)} \int_0^{x_i} \beta_i\xi\mathrm{d}\xi. \tag{11}$$