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Survey paper

On relationships among passivity, positive realness, and dissipativity in linear systems[☆]

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ABSTRACT

The notions of passivity and positive realness are fundamental concepts in classical control theory, but the use of the terms has varied. For LTI systems, these two concepts capture the same essential property of dynamical systems, that is, a system with this property does not generate its own energy but only stores and dissipates energy supplied by the environment. This paper summarizes the connection between these two concepts for continuous and discrete time LTI systems. Beyond that, relationships are provided between classes of strictly passive systems and classes of positive real systems. The more general framework of dissipativity is introduced to connect passivity and positive realness and also to survey other energy-based results. The frameworks of passivity indices and conic systems are discussed to connect to passivity and dissipativity. After surveying relevant existing results, some clarifying results are presented. These involve connections between classes of passive systems and finite-gain L_2 stability as well as asymptotic stability. Additional results are given to clarify some of the more subtle conditions between classes of these systems and stability results. This paper surveys existing connections between classes of passive and positive real systems and provides results that clarify more subtle connections between these concepts.

1. Introduction

In our recent research we have pursued constructive techniques based on passivity theory to design networked-control systems which can tolerate time delay and data loss, see e.g. Kottenstette and Antsaklis (2007a,b), Kottenstette, Hall, Koutsoukos, Sztipanovits, and Antsaklis (2012) and McCourt and Antsaklis (2012). As a result we have had to rediscover and clarify key relationships between three classes of systems. The first class is passive and strictly passive systems, which are characterized by a time-based input–output relationship, see e.g. Desoer and Vidyasagar (1975) and Zames (1966a,b). The second class is dissipative systems, which satisfy a time-based property that relates an input–output energy supply function to a state-based storage function, see e.g. Goodwin and Sin (1984), Hill and Moylan (1980) and Willems (1972a). The third class is that of positive real and strictly positive real systems, which are characterized by a frequency-based input–output relationship, see e.g. Anderson (1967), Haddad and Bernstein (1994), Hitz and Anderson (1969), Tao and Ioannou (1990) and Wen (1988b). It is noted in Willems (1972b) that, for the continuous time case, these relationships "are all derivable from the same principles and are part of the same scientific discipline". However, it is not clear that such connections have been fully exploited, although recently Haddad and Chellaboina (2008) provided an excellent exposition of some such connections. The goals of this paper are to (1) review the classical notions of passivity, dissipativity, and positive realness; (2) summarize existing relationships between these classes of systems with appropriate references; and (3) provide original results to clarify these relationships. These are broad research areas and entire surveys have been devoted to passivity and dissipativity. Rather than attempting to survey all major contributions to these fields, this paper instead reviews literature and

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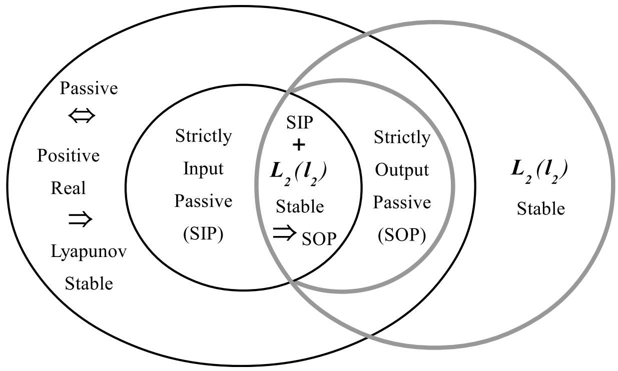

Fig. 1. This Venn Diagram shows relationships between passivity, positive realness, and L_2 stability for continuous and discrete time *LTI* systems.

results that address the relationships between these concepts in order to identify discrepancies and provide clarifying results and remarks.

While passivity and dissipativity are typically applied to general nonlinear systems, this paper focuses on the linear time invariant (LTI) case to emphasize the connection to positive real systems, as this notion only applies to LTI systems. Some of the basic results covered in this paper are summarized in Fig. 1. The foundational relationship is that, for LTI systems, the property of passivity is equivalent to the property of positive realness. Under mild technical assumptions, these systems are Lyapunov stable. For LTI systems, *strict passivity* is equivalent to strict positive realness. For asymptotically stable systems, strongly positive real is equivalent to strictly input passive (*SIP*). While the figure shows that *SOP* systems are passive and L_2^m (l_2^m) stable it should be noted that this relationship is sufficient only. Systems that are passive and L_2^m (l_2^m) stable are not necessarily *SOP*. This fact will be demonstrated with a counterexample. Another connection from Fig. 1 is that systems that are both *SIP* and L_2^m (l_2^m) stable must be *SOP*. Other relationships will be covered that relate *SIP*, strictly output passive (*SOP*), and very strictly passive (*VSP*) to notions of stability and of state strict passivity. Some preliminary results from this paper were presented in Kottenstette and Antsaklis (2010). The current paper expands on those connections and presents additional clarifying results. An application of these results to passivity-based pairing in MIMO systems can be found in Kottenstette, McCourt, Xia, Gupta, and Antsaklis (2014).

This paper is organized as follows. A brief review of some relevant literature is included in Section 2. This includes a selection of classical results that have been important to the field as well as recent results for this area. Section 3 provides definitions of the energy-based properties used in this paper. This section begins with some mathematical preliminaries and then moves on to define passivity, dissipativity, positive realness, and passivity indices. Section 4 includes some basic stability results for passivity and dissipativity and then moves into some fundamental results involving passive and positive real systems. The main results of the paper are given in Section 5. Concluding remarks are provided in Section 6.

2. Brief review of energy-based control

Passivity, dissipativity, and positive realness have had an important history in energy-based control. There have been numerous papers written on these topics as this is an important area of linear and nonlinear control. Instead of surveying the breadth of all these topics, this paper focuses on relationships between topics. The following provides a brief review of the relevant foundational works in these areas. This is followed by a survey of recent results to demonstrate the diverse use of these notions in modern control.

2.1. Classical results

The notion of passivity originated in electrical circuit theory where circuits made up of only passive components were known to be stable. It was also known that any two passive circuits could be interconnected in feedback or in parallel and the resulting circuit would still be passive, see e.g. Anderson and Vongpanitlerd (1973). This compositionality property greatly reduces the analysis required to demonstrate stability for a network of circuits. The property of passivity itself is an energy-based characterization of the input–output behavior of dynamical systems. A passive system is one that stores and dissipates energy without generating its own. The notion of stored energy can be either a traditional physical notion of energy, as it is with many physical systems, or a generalized energy, see Anderson and Vongpanitlerd (1973) and Desoer and Vidyasagar (1975). Passivity and dissipativity were formalized for general nonlinear state space systems in Willems (1972a,b). These papers provided results for passivity, specifically that passive systems were stable and that the passivity property was preserved when systems were combined in feedback or parallel. Specific forms of dissipativity for nonlinear control affine systems were studied further in Hill and Moylan (1976, 1977, 1980). Dissipativity was studied for more general nonlinear systems in continuous time in Lin (1995, 1996) and in discrete time in Lin (1996) and Lin and Byrnes (1994).

As the focus of this survey is on the relationship between passive systems and positive real systems, the Positive Real Lemma is of special importance. This is also known as the KYP Lemma which originated in Kalman (1963) using results from Popov (1961) and Yakubovich (1962). This was extended to multi-variable systems in Anderson (1967) with an alternative proof given in Rantzer (1996). Later this lemma would be used to develop linear matrix inequality (LMI) methods to demonstrate passivity for linear systems, see Boyd, El Ghaoui, Feron, and Balakrishnan (1994).

A particularly valuable survey paper, Kokotovic and Arcak (2001), covered the history of constructive nonlinear control with a focus on passivity and dissipativity. From the same time period a tutorial style paper, Ortega, van der Schaft, Mareels, and Maschke (2001), provided a strong motivation for passivity-based control and more generally energy-based control. A more recent reference highlighting advances in energy-based methods is Ebenbauer, Raff, and Allgöwer (2009). In Willems (2007), the classical work in dissipativity was reassessed from an updated perspective. Strong introductions to passivity can be found in the textbooks Khalil (2002) and van der Schaft (1999). The more general framework of dissipativity is thoroughly covered in Bao and Lee (2007), Brogliato, Lozano, Maschke, and Egeland (2007), and Haddad and Chellaboina (2008).

2.2. Recent progress

For passivity and dissipativity, progress has been made recently in numerous areas. While passivity based control has traditionally been applied to electrical circuits, see e.g. Anderson and Vongpanitlerd (1973), and robotic manipulators, see e.g. Spong, Hutchinson, and Vidyasagar (2006), recently these approaches have been expanded to chemical processes, where passivity can be used to design robust controllers as in Bao, Lee, Wang, and Zhou (2003) and Bao and Lee (2007). Passivity methods have been used in temperature control in buildings as in Mukherjee, Mishra, and Wen (2012), where the transient and steady state control performance can be improved. Another application area is in Freidovich, Mettin, Shiriaev, and Spong (2009) where passivity was used to design stable gaits for walking robots. Passivity has also been used as a design tool for coordination in multi-agent systems in Arcak (2007)