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Observability limits for networked oscillators☆

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ABSTRACT

Inspired by the neuro-scientific problem of predicting brain dynamics from electroencephalography (EEG) measurements of the brain's electrical activity, this paper presents limitations on the observability of networked oscillators sensed with quantised measurements. The problem of predicting highly complex brain dynamics sensed with relatively limited amounts of measurement is abstracted to a study of observability in a network of oscillators. It is argued that a low-dimensional quantised measurement is in fact, by itself, an exceptionally poor observer for a large-scale oscillator network, even for the case of a completely connected graph. The main rational is based on (i) an information-theoretic argument based on ideas of entropy in measure preserving maps, (ii) a linear deterministic observability argument, and (iii) a linear stochastic approach using Kalman filtering. For prediction of brain network activity, the findings indicate that the classic EEG signal is just not precise enough to be able to provide reliable prediction and tracking in a clinical setting in view of the complexity of underlying neural dynamics.

1. Introduction

Observability issues in networked oscillators have implications for tracking and control in a wide range of distributed systems that exhibit self-organising behaviour through synchronisation, from modern telecommunications (Prehofer & Bettstetter, 2005) to future power systems (Butler, 2007; Rohden, Sorge, Timme, & Witthaut, 2012). Coupled oscillatory networks are particularly prevalent in biological systems where there is increasing interest in tracking and predicting dynamics for applications in medical bionics.

Our particular motivational interest is the human brain, an oscillatory system with an estimated 86G neuron cells (Azevedo et al., 2009) networked with 1P synaptic connections.[2] EEG recordings of the brain's electrical activity typically provide our output signal from which to observe the underlying activity. Observability of the activity of neurons in the brain system from EEG measurements is essential for advancement of medical treatment through systems control in a range of neural conditions including epilepsy, Parkinson's disease and depression. Strong observability is also necessary for prediction in neurology, for example of epileptic episodes or to determine which patient are likely to respond well to classes of pharmaceutical drugs.

Current network observability analyses typically seek to exploit redundancies in the connection pathways between network nodes to determine the minimum number of sensor measurements required such that all nodes are either directly or indirectly reachable. Early work includes finding observable islands (or sub-networks) when the network as a whole is unobservable and further determining where to place additional measurements to reach network areas beyond these islands (Monticelli & Wu, 1985; Wu & Monticelli, 1985). This early work is iterative in nature and therefore computationally intractable for large-scale networks. Recent work by Liu et al. in network observability, using graphical approaches to find the minimum sensor set, does however cater for large-scale networks (Liu, Slotine, & Barabási, 2013). These methods all rely on exploiting topological clustering. By contrast, the approach in this paper considers a network with fully-connected graph, where clustering is unavailable.

Liu et al. also highlight interest in the applicability of their methods to coupled oscillator systems (Liu et al., 2013); a similar question was recently considered, in the small network case,

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[2] G denotes giga (10^9) and P denotes peta (10^{15}).

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for synchronous neuronally-inspired networks (Whalen, Brennan, Sauer, & Schiff, 2012). In this case observability was found to be quite limited and heavily influenced by topology and symmetry. In this paper we consider observability in large scale coupled oscillator systems.

Liu and Bitmead consider network observability in non-linear stochastic networks, defining observability in information theoretic terms by comparing the entropy of the state with the conditional entropy of the state given the measurement (Liu & Bitmead, 2011). Although stochastic systems are not considered in this paper, similar ideas of linking observability to entropy are applied here (although it is entropy of the state map that is considered in this work rather than a condition related to the mutual information between state and measurement signal). In both cases, however, positive entropy effectively allows for a reduction of state uncertainty with an increasing sequence of measurements. As articulated by Liu and Bitmead, the power in an entropy definition of observability is that it can apply to both linear and non-linear systems.

The Takens–Aeyels embedding theorem (Aeyels, 1981; Takens, 1981) states that observability is a generic property in non-linear (autonomous) systems. Moreover, a state may be reconstructed from the measurement vectors for sufficiently large measurement time series. This state can then be used to infer dynamics. This remarkable theorem of delay reconstruction provides an elegant and resourceful tool, but, it is limited to autonomous, stationary, noise-free systems (Kantz & Schreiber, 2004, Chapter 3). Despite such limitations, delay reconstruction is widely applied to real-world systems, for example to the brain (Iasemidis, Sackellares, Zaveri, & Williams, 1990; Lehnertz & Elger, 1998; Le Van Quyen et al., 2001) where dynamics are certainly not noise-free or indeed autonomous and can only be considered quasi-stationary on short time scales (∼10 s) (Niedermeyer & Lopes Da Silva, 2005). How realistic is the Takens–Aeyels embedding theorem here, particularly in light of such a large-scale system as the brain?

In this paper the question of what one can observe from an EEG record is reduced to the generalised question of how quantisation of an output, from a large-scale system, effects the observability. This effect of quantisation has implications beyond the reconstruction of brain dynamics for the observability of any practical system.

A "synthetic" brain-like situation is presented in Section 2, fully under our control, within which the limits of a brain-like EEG recording can be investigated. While Section 3 demonstrates that we have a theoretically observable system, practical considerations reveal a severe lack of observability using arguments from (i) information-theoretic ideas of entropy in measure preserving maps in Section 4.1, (ii) linear deterministic observability in Section 4.2 and (iii) linear stochastic Kalman filtering in Section 4.3. The implications for prediction and tracking of brain dynamics are discussed in Section 5 followed by concluding comments in Section 6.

2. Abstraction to networked clocks

A generic and scalable coupled oscillator model (with origins in the 1985 work of Wright, Kydd, & Lees, 1985) is proposed as a synthetic brain. It is important to highlight that this is not a model which can tell us anything about the nature of brain function, but this approach is suited, however, to the specific task of investigating what underlying information one may expect to recover from the EEG signal.

The model abstracts the problem to the study of a simple network of second order oscillators with linear interconnection. Such a model neglects the complexities of biologically realistic neuron-dynamics and instead formulates the observability problem as a generic network of oscillators where an EEG-like measurement is made. The model is scalable in the sense that the size and location of the recording electrode and the domain of tissue that is measured are immaterial—it can equally represent the measurement of a localised region of brain tissue from an implanted microelectrode to measurement of a large area of brain tissue from the scalp surface.

EEG recordings are modelled as the output (defined as a linear map from the state) of a system of networked oscillators. The use of linear observation is entirely biologically appropriate (Varsavsky, Mareels, & Cook, 2010, Chapter 2). Each individual oscillator is modelled as a pendulum clock,

$$\ddot{x}_i + \omega_i^2 \sin(x_i) = F_i, \quad i = 1 \ldots N, \tag{1}$$

where x_i is angular position, ω_i is the natural frequency of oscillation and F_i is the forcing term of the ith pendulum defined as $F_i = \gamma_i \sin(\omega_{in,i} t) + \sum_j \alpha_{ij}(x_j - x_i)$. F_i consists of an external input term, $\gamma_i \sin(\omega_{in,i} t)$, plus a feedback term that couples the position state from other pendula, $\sum_j \alpha_{ij}(x_j - x_i)$. $\omega_{in,i}$ is the frequency of external input and γ_i denotes the strength of external input to oscillator i. α_{ij} denotes the coupling strength between oscillators i and j. Examples of external inputs are sensory input to a large brain network, input from distant regions of the brain that are external to a localised model and the therapeutic electrical brain stimulation discussed in Freestone et al. (2011) and Nelson et al. (2011).

Consider the linearisation of (1) for simplicity,

$$\ddot{x}_i + \omega_i^2 x_i = F_i, \quad i = 1 \ldots N, \tag{2}$$

where these linear clocks can equally be combined to form a clock network through coupling.

The brain network exhibits small world characteristics with locally dense clusters and a hop number of less than 3 (Achard, Salvador, Whitcher, Suckling, & Bullmore, 2006; Crick & Jones, 1993). For this work a simple fully-connected graph is assumed, with strong local connection and weaker long range connection, as illustrated in Fig. 1. Additionally, it is assumed that all model parameters are known (a huge simplification from the true brain/EEG observability problem). It is chosen to restrict the model to pure or marginally stable oscillators rather than include a damping term. This is biologically justifiable given the alternatives to marginal stability are either (i) damped oscillation or (ii) unstable oscillation. For the electro-magnetic activity of the brain (i) and (ii) would indicate pathological states of activity, for example decay of neural activity until brain death and increasing neural excitability until an epileptic seizure state respectively. Therefore, operating the coupled oscillator model on the boundary of stability is bio-realistic for normal brain function. While certainly the brain will naturally deviate into both slightly underdamped and slightly damped modes of oscillation, it is envisioned that the natural balance of excitability in the brain will constrain this damped and underdamped activity to dynamics that lie near to the critically damped boundary.

A coupled clock network of N linear clocks (illustrated for $N = 4$ in Fig. 2) can be written in state space format as $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$ with an ideal EEG measurement of $\mathbf{y} = \mathbf{Cx}$ using ($\mathbf{A}$, $\mathbf{C}$, $\mathbf{B}$) defined in (3), (7), (8). $\mathbf{x}$ is the state and $\mathbf{u}$ represents an input given in Eq. (9).

$$\mathbf{A}(\omega) = \begin{bmatrix} \mathbf{A}_1 + \varepsilon_{11} & \varepsilon_{12} & \cdots & \varepsilon_{1N} \\ \varepsilon_{21} & \mathbf{A}_2 + \varepsilon_{22} & \cdots & \varepsilon_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{N1} & \varepsilon_{N2} & \cdots & \mathbf{A}_N + \varepsilon_{NN} \end{bmatrix}, \tag{3}$$

where

$$\mathbf{A}_i = \left((0, +\omega_i)', (-\omega_i, 0)'\right), \tag{4}$$