Automatica 50 (2014) 1100–1108

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Controlling anonymous mobile agents with unidirectional locomotion to form formations on a circle[☆]

Chen Wang[a,b], Guangming Xie[a,1], Ming Cao[b]

[a] Intelligent Control Laboratory, College of Engineering, Peking University, 100871 Beijing, China

[b] Faculty of Mathematics and Natural Sciences, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands

ARTICLE INFO

Article history:
Received 2 August 2012
Received in revised form
15 August 2013
Accepted 24 January 2014
Available online 12 March 2014

Keywords:
Circle formation
Multi-agent system
Distributed control
Sampled-data control
Unidirectional locomotion

ABSTRACT

We study the circle forming problem in which a group of anonymous mobile agents are required to form a formation when moving on a given circle. The agents are constrained to move in the one-dimensional space of the circle only in the counterclockwise direction, but not the opposite way. Distributed, cooperative, sampled-data control strategies are designed that only take nonnegative values. We prove that the multi-agent system under such constrained control input can be guided to reach the prescribed circle formation asymptotically with the additional guarantee that no collision between agents ever takes place. The theoretical analysis is further validated through simulations.

1. Introduction

When using a team of autonomous mobile robots to carry out environmental monitoring or exploration tasks, it is sometimes advantageous to keep the robots moving in a formation with certain geometric shapes in order to improve the quality of the data collected collectively or the robustness of group motion against random environmental disturbances (Dhariwal, Sukhatme, & Requicha, 2004; Leonard et al., 2007; Roy & Dudek, 2001). The challenge for such pattern-forming problems for robotic teams lies in the fact that the robots usually cannot rely on centralized coordination and have to use local information to implement their distributed control strategies. Forming circle formations becomes a benchmark problem, since on one hand circle formations are one of the simplest classes of formations with geometric shapes and on the other they are natural choices of the geometric shapes for a robotic team to exploit an area of interest (Krick, Broucke, & Francis, 2009; Leonard et al., 2010; Suzuki & Yamashita, 1999).

The algorithmic study on the problem of forming circle formations is rooted in the study on multi-agent systems in theoretical computer science. The *semi-synchronous* model, developed in Suzuki and Yamashita (1999) and later on generalized in Chatzigiannakis, Markou, and Nikoletseas (2004); Défago and Konagaya (2002); Défago and Souissi (2008), has been successful in gaining insight into the key characteristics of the autonomous agents that make the circle forming problem challenging. Such characteristics restrict the agents' recognition, sensing and communication capabilities and the relaxation of any of them will make the problem formulation less appealing. To be more specific, the agents are assumed to be oblivious (without memories about past actions and observations), anonymous (not distinguishable from one another), unable to communicate directly, and can only interact through sensing other agents' positions. Using the semi-synchronous model, Défago and Konagaya (2002) have proposed algorithms to decompose the circle formation problem into two subproblems and solve them separately. The first is to form a circle in finite time, and the second is to guide the robots to the configuration where all of them are positioned evenly on the circle. Such algorithms are simplified in Chatzigiannakis et al. (2004) and unified in Défago and Souissi (2008). The assumptions in Défago and Konagaya (2002) are also used in Chatzigiannakis et al. (2004) and Défago and Souissi (2008).

People have worked on designing distributed control laws for teams of anonymous mobile agents to realize circle formations

☆ This work was supported in part by grants from the National Natural Science Foundation of China (NSFC, No. 60774089, 10972003), the Dutch Organization for Scientific Research (NWO), the Dutch Technology Foundation (STW) and the European Research Council (ERC, ERC-StG-2012-307207). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Antonis Papachristodoulou under the direction of Editor Frank Allgöwer.

E-mail addresses: wangchen@pku.edu.cn (C. Wang), xiegming@pku.edu.cn (G. Xie), m.cao@rug.nl (M. Cao).

[1] Tel.: +86 10 62754083; fax: +86 10 62754388.

http://dx.doi.org/10.1016/j.automatica.2014.02.036
0005-1098/

where agents are evenly distributed on a circle (Bullo, Cortes, & Martinez, 2009; Marshall, Broucke, & Francis, 2004). Only a few works (Wang, Xie, & Cao, 2013) have considered the problem of forming arbitrary formations on a circle. There is even fewer work (Elor & Bruckstein, 2011; Flocchini, Prencipe, & Santoro, 2008) has considered locomotion constraints for the implementation of such control laws, while such constraints sometimes are critical characterizations of mobile agents. In particular, in this paper besides the agents' characteristics considered in the semi-synchronous model, we further impose the locomotion constraint that the mobile agents can only move forward but not backward. This characterization is motivated by several types of mobile robots. The well-known Dubin's vehicles can only move forward following trajectories with bounded curvatures (Dubins, 1957); some biomimetic robots are mechanically prohibited to move backward, such as robotic fish (Wang, Xie, Wang, & Cao, 2011) and salamander (Ijspeert, Crespi, Ryczko, & Cabelguen, 2007).

The goal of this paper is to design distributed control laws that can guide a group of autonomous mobile agents to form any given circle formations by moving unidirectionally on the prescribed circle. We consider a system consisting of multiple mobile agents modeled by point masses, all of which move in the one-dimensional space of the given circle. The agents are oblivious, anonymous, and unable to communicate directly; they share the common notion of being clockwise on the circle. Each agent can only sense the relative angular positions of its neighboring two agents that are immediately in front of or behind itself. The agents can only move counterclockwise on the circle.

Considering the applications in real-robot systems, we propose to use sampled-data control laws since continuous-time control laws may be difficult to be implemented directly because of hardware constraints, such as communication bandwidth, rise time, and computational load (Chen & Francis, 1995). Although input-constrained systems are in general difficult to analyze, we follow a novel approach that translates the unidirectional locomotion constraints into the time-varying neighbor relationships between agents. Consequently, we are able to obtain the closed-loop system dynamics in the form of time-varying, switched, linear, discrete-time systems. The technical challenge is then how to prove convergence for such systems. Using tools from matrix analysis, we show that not only the agents achieve prescribed circle formations asymptotically, but the spatial ordering of the agents is preserved throughout the evolution as well. The main contribution of the paper is then the methodology to deal with unidirectional locomotion constraints for cooperative tasks of forming circle formations for teams of anonymous mobile agents. Note that kinematic point models for mobile agents have been popular in formation control literatures. They are ideal for the scenarios when the scale of the areas, where the agents are deployed, is significantly greater than the physical dimensions of the agents.

We emphasize that a recent paper (Elor & Bruckstein, 2011) by Elor and Bruckstein in theoretical computer science has presented interesting impossibility results and discrete-time algorithms for the uniform circle formation problem when the agents move only in one direction on the vertices of a ring graph. A similar continuous version of the problem has been discussed in an earlier paper (Flocchini et al., 2008) when the agents move in the continuous one-dimensional space of the circle. Such theoretical computer science papers focus more on the algorithmic aspects of the solutions to the circle formation problem, e.g. the existence and complexity of an algorithm. As a result, there is usually no dynamic model for the agents and the agents execute the designed algorithms in an open-loop fashion in that they are assumed to always be able to move to their waypoints after some finite time. In comparison, we formulate dynamical system models for the multi-agent systems; more specifically, we have assumed that the agents' dynamics can be described by single-integrators and we study closed-loop

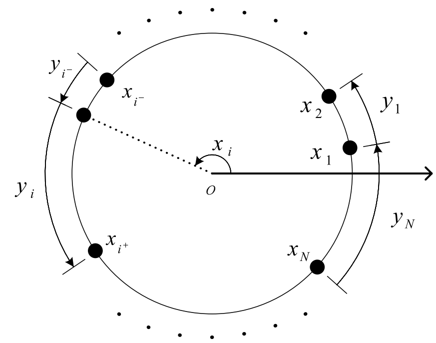

Fig. 1. Agents distributed on a circle.

systems since sampled angular position information is used in the proposed control laws. A critical assumption in Elor and Bruckstein (2011) and Flocchini et al. (2008) is that each agent operates in the look–compute–move–wait cycles and that in a cycle each agent always moves to its calculated waypoint before waiting to start a new cycle. Such an assumption is not needed in our paper. So the methodologies, focuses and analysis of the work done in Elor and Bruckstein (2011) and Flocchini et al. (2008) are quite different from what we present in this paper. Moreover, since their works mainly focus on the existence of an algorithm to solve the circle formation problem under specific assumptions made by theoretical computer scientists, it is naturally important to see how to use tools from sampled-data control theory to make the problem solvable with modifications in the assumptions.

The rest of the paper is organized as follows. In Section 2, we formulate the circle formation problem. Then we propose a sampled-data distributed control law in Section 3 and analyze its performances in Section 4. Simulation results are given in Section 5.

2. Problem formulation

We consider a group of $N, N \geq 2$, agents that are initially positioned on a given circle. They share the common knowledge about the direction of being clockwise on the circle and are constrained to move only counterclockwise. Although the agents are anonymous and they cannot distinguish one from another, for ease of expression, we label the agents counterclockwise by $1, 2, \ldots, N$ as shown in Fig. 1. In a fixed coordinate system of choice, let $x_i(t)$ be the position of agent i, $1 \leq i \leq N$, at time t measured by angles and without loss of generality assume that the agents do not coincide in the beginning and their initial positions satisfy

$$0 \leq x_1(0) < \cdots < x_i(0) < x_{i+1}(0) \cdots < x_N(0) < 2\pi. \quad (1)$$

Note that for the same angle traveled by an agent, different radii of the given circle pose different requirement on the actuation capabilities of the agents. We consider the case when the agents' neighbor relationships are described by a ring $\mathbb{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ and $\mathcal{E} = \{(1, 2), (2, 3), \ldots, (N-1, N), (N, 1)\}$; in other words, each agent can only sense the relative positions of the two agents that are immediately in front of or behind itself. Note that by using the information about the two immediate neighbors, the oblivious agents do not need to track who its neighbors are; in fact, as to be proved later in the paper, under our control laws, the neighbor relationships are guaranteed to be fixed, which greatly simplifies the analysis. We denote the set of agent i's two neighbors by $\mathcal{N}_i = \{i^-, i^+\}$ where

$$i^+ = \begin{cases} i+1 & \text{when } i = 1, 2, \ldots, N-1 \\ 1 & \text{when } i = N, \end{cases}$$