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Brief paper

The leader-following attitude control of multiple rigid spacecraft systems ☆

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ABSTRACT

In this paper, we consider the leader-following consensus problem for a multiple rigid spacecraft system whose attitude is represented by the unit quaternion. Most results on this problem rely on the assumption that every follower can access the state of the leader and are obtained via a decentralized control manner. By developing a nonlinear distributed observer for the leader system, we can solve this problem via a distributed control scheme under the mild assumptions that the state of the leader can reach every follower through a path and that the communication between followers is bidirectional. Moreover, our result can accommodate a class of desired angular velocities generated by a marginally stable linear autonomous system.

1. Introduction

Attitude tracking of spacecraft systems has been a benchmark control problem and has been extensively studied under various scenarios in, say, Ahmed, Coppola, and Bernstein (1998), Chen and Huang (2009), Luo, Chu, and Ling (2005), Wen and Kreutz-Delgado (1991), Wie (1998), Yoon and Tsiotras (2002) and Yuan (1988). More recently, much attention has been paid to the problem of attitude consensus for a group of spacecraft systems (Scharf, Hadaegh, & Ploen, 2004), which can be classified as either leaderless consensus problem or leader-following consensus problem. The leader-following attitude consensus problem aims to design a control law such that the attitude of each follower subsystem will asymptotically track a prescribed trajectory while the leaderless attitude consensus problem only requires the control law to drive the attitude of each subsystem to a common trajectory.

The leaderless attitude consensus problem of a multiple spacecraft system has been studied in a few papers (Lawton & Beard, 2002; Ren, 2007a; Sarlette, Sepulchre, & Leonard, 2009). While, the leader-following case is more interesting in the sense that it has full command of the asymptotic behavior of all spacecraft. The leader-following attitude consensus problem can be studied by two schemes, i.e., the decentralized control scheme and the distributed control scheme. The decentralized scheme relies on the assumption that every follower can access the state of the leader, or what is the same, the desired attitude and angular velocity. Such a scheme has been employed in, say, Abdessameud and Tayebi (2009), Chen, Ji, and Bi (2009), Nair and Leonard (2007) and (VanDyke & Hall, 2006). In contrast, the distributed control scheme does not require the state of the leader to be accessible to all followers and will make use of some type of cooperation among the controllers of all follower subsystems. Typical results on the leader-following attitude formation problem via distributed control can be found in Bai, Arcak, and Wen (2008), Ren (2007a,b). However, in Ren (2007a), the leader-following consensus was achieved only for the special case where the desired attitude is constant and the desired angular velocity is zero. In Bai et al. (2008), while the angular velocity of all followers can converge to a class of time-varying desired angular velocity, the asymptotic behavior of the attitudes of followers will rely on their initial conditions. In Ren (2007b), the angular velocity of all followers can also converge to a class of time-varying desired angular velocity, but the leader-following attitude consensus is only achieved if "the information exchange topology can be simplified to a directed graph with only one node".

In this paper, we will further consider the leader-following attitude consensus problem for a multiple rigid spacecraft system

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without assuming that every follower can access the state of the leader. The control scheme consists of two steps. First, for each follower system, a nonlinear distributed observer is developed to estimate the state of the leader system, under the mild assumptions that the state of the leader can reach every follower through a path and that the communication between followers is bidirectional. Next, the control input is synthesized based on the state of the observer and the plant. Our result has two features. First, we introduce a marginally stable linear system to generate the desired angular velocity. This scheme enables our control law to handle a class of reference trajectories including step signal with arbitrary amplitude, sinusoidal signal with arbitrary amplitude and initial phase and the combination of the step signal and the sinusoidal signal. Second, our control law achieves both attitude and angular velocity tracking.

The rest of this paper is organized as follows. In Section 2, we formulate the problem. In Section 3, a nonlinear distributed observer is given and based on it, a distributed control law is proposed in Section 4 to solve the problem defined in Section 2. In Section 5, we present simulation results to validate the effectiveness of our control law. Finally, we close the paper with some concluding remarks in Section 6.

In what follows, we use the following notations. $\otimes$ denotes the Kronecker product of matrices. 1_N denotes an N dimensional column vector whose components are all 1. $\|x\|$ denotes the Euclidean norm of vector x and $\|A\|$ denotes the Euclidean norm of matrix A. For $x_i \in R^{n_i}, i = 1, \ldots, m$, $\mathrm{col}(x_1, \ldots, x_m) = [x_1^T, \ldots, x_m^T]^T$. For $x = \mathrm{col}(x_1, x_2, x_3) \in R^3$, define

$$x^\times = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}.$$

It can be verified that $x^T x^\times = 0$.

2. Problem formulation

In this paper, we consider a group of N rigid spacecraft systems as follows:

$$\dot{\hat{q}}_i = \frac{1}{2}\hat{q}_i^\times \omega_i + \frac{1}{2}\bar{q}_i\omega_i, \qquad \dot{\bar{q}}_i = -\frac{1}{2}\hat{q}_i^T\omega_i \tag{1a}$$

$$J_i\dot{\omega}_i = -\omega_i^\times J_i\omega_i + u_i, \quad i = 1, \ldots, N \tag{1b}$$

where $q_i = \mathrm{col}(\hat{q}_i, \bar{q}_i)$ with $\hat{q}_i \in R^3, \bar{q}_i \in R$ is the unit quaternion expression of the attitude of the body fixed frame $\mathcal{B}_i$ of the ith spacecraft system relative to the inertial frame $\mathcal{I}$. $\omega_i \in R^3$ is the angular velocity of $\mathcal{B}_i$ relative to $\mathcal{I}$; $J_i \in R^{3\times 3}$ is the positive definite inertia matrix; $u_i \in R^3$ is the control torque. ω_i, J_i and u_i are all expressed in $\mathcal{B}_i$.

In addition, we assume that the desired attitude q_0 and the desired angular velocity ω_0 of system (1) are generated by the following system

$$\dot{\hat{q}}_0 = \frac{1}{2}\hat{q}_0^\times \omega_0 + \frac{1}{2}\bar{q}_0\omega_0, \qquad \dot{\bar{q}}_0 = -\frac{1}{2}\hat{q}_0^T\omega_0 \tag{2a}$$

$$\dot{\omega}_0 = S\omega_0 \tag{2b}$$

where $q_0 = \mathrm{col}(\hat{q}_0, \bar{q}_0)$ with $\hat{q}_0 \in R^3, \bar{q}_0 \in R$ represents the attitude of the leader frame $\mathcal{B}_0$ relative to the inertial frame $\mathcal{I}$, $\omega_0 \in R^3$ is the angular velocity of $\mathcal{B}_0$ relative to $\mathcal{I}$, expressed in $\mathcal{B}_0$, and $S \in R^{3\times 3}$ is a constant matrix.

Remark 2.1. It is noted that q_0 is generated by the same system as that in Ren (2007b) and Yuan (1988) while ω_0 is generated by a linear autonomous system which, under Assumption 2, can generate a class of signals such as the step function, sinusoidal functions of various frequencies and their finite many combinations.

Remark 2.2. The attitude consensus problem has also been studied with the attitude being represented by the modified Rodrigues parameters (MRPs) (Chung, Ahsun, & Slotine, 2009; Dimarogonas, Tsiotras, & Kyriakopoulos, 2009; Du, Li, & Qian, 2011; Ren, 2010; Zou, Kumar, & Hou, 2012). With the MRPs representation, the attitude dynamic equation can be converted into the Lagrangian form and many classical approaches can be used to solve the attitude consensus problem, such as adaptive control or sliding mode control. However, the MRPs representation is not global due to the singularity and thus we choose to represent the attitude of the spacecraft in unit quaternion in this paper.

The system composed of (1) and (2) can be viewed as a multi-agent system of $(N + 1)$ agents with (2) as the leader and the N subsystems of (1) as N followers. Given (1) and (2), we can define a graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$[2] with $\bar{\mathcal{V}} = \{0, 1, \ldots, N\}$ and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. Here the node 0 is associated with the leader system (2) and the node i, $i = 1, \ldots, N$, is associated with the ith subsystem of the follower system (1). For $i = 0, 1, \ldots, N, j = 1, \ldots, N$, $(i, j) \in \bar{\mathcal{E}}$ if and only if u_j can use the full state of agent i for control. Let $\bar{\mathcal{N}}_i$ denote the neighbor set of the node i of $\bar{\mathcal{G}}$. We can further define a subgraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of $\bar{\mathcal{G}}$ where $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is obtained from $\bar{\mathcal{E}}$ by removing all the edges between node 0 and the nodes in $\mathcal{V}$.

In terms of $\bar{\mathcal{G}}$, we can describe a distributed control law as follows, for $i = 1, \ldots, N$,

$$u_i = k_i(q_i, \omega_i, \varphi_i) \tag{3a}$$

$$\dot{\varphi}_i = g_i(\varphi_i, \varphi_j - \varphi_i, j \in \bar{\mathcal{N}}_i) \tag{3b}$$

where k_i and g_i are smooth functions, and $\varphi_0 = \mathrm{col}(q_0, \omega_0)$.

To introduce our problem, like in Chen and Huang (2009) and Sidi (1997), we perform on systems (1) and (2) the following transformation:

$$\hat{\epsilon}_i = \bar{q}_0\hat{q}_i - \hat{q}_0^\times\hat{q}_i - \bar{q}_i\hat{q}_0 \tag{4a}$$

$$\bar{\epsilon}_i = \hat{q}_i^T\hat{q}_0 + \bar{q}_i\bar{q}_0 \tag{4b}$$

$$\hat{\omega}_i = \omega_i - C_i\omega_0 \tag{4c}$$

where $C_i = (\bar{\epsilon}_i^2 - \hat{\epsilon}_i^T\hat{\epsilon}_i)I_3 + 2\hat{\epsilon}_i\hat{\epsilon}_i^T - 2\bar{\epsilon}_i\hat{\epsilon}_i^\times$ is called the direction cosine matrix, which represents the relative attitude between $\mathcal{B}_i$ and $\mathcal{B}_0$. Let $\epsilon_i = \mathrm{col}(\hat{\epsilon}_i, \bar{\epsilon}_i)$, which is the unit quaternion representation of C_i. We have

$$\dot{\hat{\epsilon}}_i = \frac{1}{2}\hat{\epsilon}_i^\times\hat{\omega}_i + \frac{1}{2}\bar{\epsilon}_i\hat{\omega}_i, \qquad \dot{\bar{\epsilon}}_i = -\frac{1}{2}\hat{\epsilon}_i^T\hat{\omega}_i \tag{5a}$$

$$J_i\dot{\hat{\omega}}_i = -\omega_i^\times J_i\omega_i + J_i(\hat{\omega}_i^\times C_i\omega_0 - C_i\dot{\omega}_0) + u_i. \tag{5b}$$

Remark 2.3. By Proposition 1 of Yuan (1988), $\mathcal{B}_i$ and $\mathcal{B}_0$ coincide if and only if $\hat{\epsilon}_i = 0$.

We now state our problem as follows.

Problem 1. Given systems (1), (2) and the graph $\bar{\mathcal{G}}$, design a control law of the form (3) such that, for $i = 1, \ldots, N$,

$$\lim_{t\to\infty} \hat{\epsilon}_i(t) = 0 \quad \text{and} \quad \lim_{t\to\infty} \hat{\omega}_i(t) = 0$$

for all $\omega_i(0)$ and all $q_i(0)$ satisfying $\|q_i(0)\| = 1$.

Remark 2.4. If we assume that every follower can receive the state from the leader directly, then, it can easily be verified by Wen and

[2] See the Appendix for a summary of graph.