Automatica 50 (2014) 1128–1134

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Robust stabilizer design for linear time-varying internal model based output regulation and its application to an electrohydraulic system[☆]

Xingyong Song[a], Yu Wang[b], Zongxuan Sun[c,1]

[a] Innovation and Research Center, Halliburton Energy Corporation, Houston, TX 77032, United States
[b] Advanced Servo Integration, Seagate Technology LLC, Bloomington, MN 55435, United States
[c] Department of Mechanical Engineering, University of Minnesota, Minneapolis, MN 55455, United States

ARTICLE INFO

Article history:
Received 2 November 2012
Received in revised form
24 July 2013
Accepted 20 December 2013
Available online 26 February 2014

Keywords:
Time-varying output regulation
Time-varying repetitive control
LPV system stabilization
Internal model principle

ABSTRACT

This paper focuses on the design of a low order robust stabilizer for the tracking/disturbance rejection problem based on the internal model principle in the time-varying setting and its application to the hydraulic pressure tracking with varying frequency. The problem of this kind known as output regulation generally consists of two major parts: internal model unit construction and stabilizer design. While the construction of the time-varying internal model unit is non-trivial by itself and a very recent research outcome enables its synthesis for a class of linear time-varying systems, the effective stabilization of the augmented system (internal model unit and plant) for practical applications remains a challenge. This is due to the need to stabilize the high order time-varying augmented system using a low order stabilizer in a robust fashion and with desirable transient performance. While directly applying the stabilization approaches for a general LTV system will result in a high order stabilizer, a new method is proposed in this paper that overcomes this bottleneck by taking advantage of the unique structure of the internal model based control system. Instead of using a dynamic stabilizer with high order, this approach uses a sequence of time-varying gains that are directly injected into the internal model unit. A critical issue addressed is how to avoid the non-convex optimization associated with the time-varying gain synthesis and then convert the stabilizer design into a series of Linear Matrix Inequalities (LMIs). The proposed control approach is then demonstrated on an electrohydraulic system.

1. Introduction

It has been generally recognized that output regulation based on the internal model principle (Fancis & Wonham, 1976; Isidori & Byrnes, 1990) is a powerful tool for the reference tracking/disturbance rejection of a class of signals generated by an exogenous system (exo-system). While it has been extensively studied in linear time invariant (LTI) case, the extension in linear time-varying (LTV) setting is not trivial and the LTV internal model construction by itself has long been a hard problem. Recently, a novel design methodology for the construction of a time-varying internal model is proposed (Zhang & Sun, 2010) for a class of LTV systems, including many application driven physical systems. The effectiveness of this research outcome projects the potential to implement the internal model principle based output regulation in real time control experiments.

However, to implement the time-varying output regulation, once an appropriate internal model unit is formed, the tracking/disturbance rejection is then converted to a stabilization problem. So far very limited research efforts have been spent in the stabilizer design for the LTV case. One of the reasons is due to the previously unavailable time-varying internal model unit construction, which is the prerequisite for the stabilizer design. In this paper, with the internal model unit constructed from Zhang and Sun (2010), we will focus on the effective stabilizer design.

The problem considered is to have a controller to stabilize the entire augmented system formed by the internal model unit and

---

[☆] The material in this paper was partially presented at the 2012 American Control Conference (ACC 2012) (Song, Wang, & Sun, 2012), June 27–29, 2012, Montréal, Canada. This paper was recommended for publication in revised form by Associate Editor Kyung-Soo Kim under the direction of Editor Toshiharu Sugie. More systematic derivation of the proposed control methodology, detailed dynamics modeling/controller design for an electro-hydraulic system, together with more comprehensive experimental results are presented in this paper. The work is supported in part by NSF under grant CMMI-1150957.

E-mail addresses: xingyong.song@halliburton.com (X. Song), kevin.y.wang@seagate.com (Y. Wang), zsun@umn.edu (Z. Sun).

[1] Tel.: +1 612 625 2107; fax: +1 612 625 6069.

http://dx.doi.org/10.1016/j.automatica.2014.02.005
0005-1098/© 2014 Elsevier Ltd. All rights reserved.

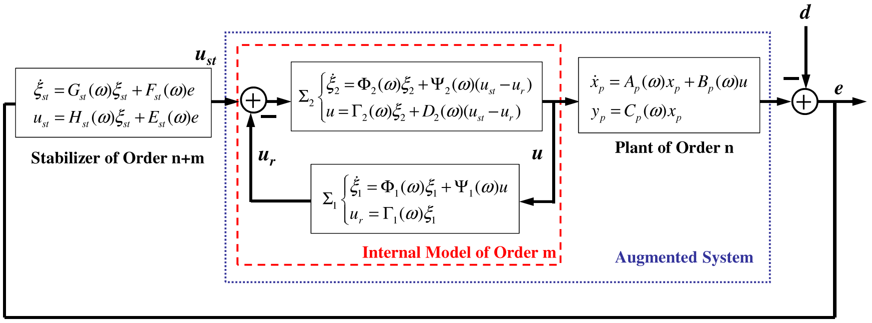

Fig. 1. Time-varying internal model and dynamic stabilizer.

the plant as shown in Fig. 1 (ω is the time-varying parameter vector). Note that both the plant and the designed internal model are time-varying. Unlike the LTI case where stabilizing the plant will automatically stabilize the entire system (Tomizuka, Tsao, & Chew, 1989), the LTV setting requires the stabilizer design to be based on the augmented system (internal model and plant) (Fig. 1). This often leads to a high order time-varying stabilizer, which is difficult to implement in real-time. Few LTV stabilization options are available, and most of the existing approaches result in a dynamic stabilizer as shown in Fig. 1 by directly applying the stabilization techniques for a general LTV system. Zhang and Sun (2010) applies the time-varying pole-placement for the stabilizer design, which is sensitive to model uncertainties and limits further experimental studies. Another route is to resort to the linear parameter varying (LPV) robust control techniques (Rugh & Shamma, 2000). However, directly applying the standard robust LPV techniques (Rugh & Shamma, 2000) to the augmented system will result in a high order stabilizer, which is not only computationally intensive for off-line synthesis but also difficult to implement in real-time.

Therefore, a low order yet robust stabilizer design for the high order time-varying internal model based control is critical, but currently is still unavailable. This fact drastically limits the experimental studies in this area. To address this problem and leverage the fact that part of the augmented system (the internal model unit) is not a physical plant and all of its states are accessible, we propose a new robust stabilizer design method. Instead of using a dynamic stabilizer with high order, we propose to use the parameter dependent output feedback gains to stabilize the system. The gain dependent control signals are directly injected into the internal model unit. By properly selecting the location of the control inputs injection inside the internal model dynamics, a series of linear matrix inequalities (LMIs) can be formulated and the parameter dependent gains can be synthesized through convex optimization. The effectiveness of the proposed approach will be shown by tracking a cyclic pressure profile with time-varying frequency on an electro-hydraulic dynamometer system.

The rest of the paper is organized as follows. Section 2 introduces the preliminaries. Section 3 presents the stabilizer design. The experimental results and application discussions will be shown in Section 4. Finally, conclusion is given in Section 5.

2. Preliminaries

2.1. Problem formulation

Consider the tracking control problem for an LTV plant of the form:

$$\begin{aligned} \dot{x}_p &= A_p(\omega)x_p + B_p(\omega)u \\ y_p &= C_p(\omega)x_p \\ e &= y_p - d \end{aligned} \tag{1}$$

where x_p belongs to R^n, the output y_p, the control input u and the regulated error e belong to R, and A_p, B_p, C_p are bounded and smooth. ω is the vector containing all the time-varying parameters. The signal d to be tracked or rejected is generated by an LTV exo-system (generating dynamics) of the form:

$$\begin{aligned} \dot{\eta} &= S(\omega)\eta \\ d &= Q(\omega)\eta \end{aligned} \tag{2}$$

where the state η belongs to R^J. The plant (1) and the exo-system (2) satisfy the following assumptions:

Assumption 1. The pair (A_p, B_p) and the pair (A_p, C_p) are uniformly controllable and observable.

Assumption 2. The exo-system (2) is stable in the sense of Lyapunov in both forward and backward direction along the time axis (Zhang, Sun, & Ye, 2010). (This property imposes that d is persistent in time while still bounded. In addition, the pair (S, Q) is uniformly observable.)

The system is successfully regulated if the following are satisfied:

1. The origin of the closed loop unforced system is a uniformly asymptotically stable equilibrium.
2. Trajectories of the closed loop system starting from any initial conditions are bounded and satisfy $\lim_{t\to\infty} e(t) = 0$.

The solution of the problem above involves two parts Zhang and Sun (2010):

Part 1 Design a time-varying internal model as shown in Fig. 1.
Part 2 Design an error feedback compensator to stabilize the augmented system.

2.2. Time-varying internal model unit construction

In this paper, the method proposed in Zhang and Sun (2010) is adopted for the internal model design. The internal model dynamics (Fig. 1) can be written as:

$$\begin{aligned} \begin{pmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{pmatrix} &= \begin{pmatrix} \Phi_1(\omega) - \Psi_1(\omega)D_2(\omega)\Gamma_1(\omega) & \Psi_1(\omega)\Gamma_2(\omega) \\ -\Psi_2(\omega)\Gamma_1(\omega) & \Phi_2(\omega) \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} \\ u &= \begin{pmatrix} -\Gamma_1(\omega) & 0 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}. \end{aligned} \tag{3}$$

Here we assume that the internal model (3) is of order m, ξ_1 belongs to R^{m-h}, and ξ_2 belongs to R^h. Usually m is much larger than the plant order n to ensure precise tracking and also to embed a wide class of generating dynamics (Zhang & Sun, 2010). Once the internal model is available, the critical problem left is to design a robust and low order stabilizer for the entire augmented system including the internal model (3) and the plant (1), which will be addressed in this paper.